Demand Deposit Contracts and Bank Runs with Present Biased Preferences

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Abstract

This paper extends the Diamond–Dybvig model of bank runs by incorporating hyperbolic discounting. The main question is how consumers’ myopic decisions affect the possibility of a bank run and the bank’s optimal contract. Under hyperbolic discounting, consumers’ deposit preferences differ from their withdrawal preferences. Therefore, the bank needs to consider two separate preferences when designing the optimal banking contract, making it more difficult to design a run-safe banking contract. This difference in preferences could increase the possibility of a bank run in equilibrium. Although the bank can design a run-proof contract, the ex-ante welfare through banking services will be lower under hyperbolic discounting due to its tighter incentive compatibility constraint.

Keywords: Bank Runs, Demand Deposits, Hyperbolic Discounting, Financial Fragility, Present Bias

JEL classifications: G21; E44; D53

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1. Introduction

This paper analyzes how consumers’ present bias (i.e., hyperbolic discounting) affects demand deposit contracts and the possibility of bank runs. We incorporate hyperbolic discounting theory into the bank-run model developed by Diamond and Dybvig (1983) and Peck and Shell (2003). The main contribution of this paper is to show that present bias can be a source of financial fragility. Many studies have suggested that cognitive bias might be a main cause of a financial crisis and financial fragility (Gennaioli and Shleifer 2018). Empirical evidence also indicates that financial crises and financial fragility cannot be explained only by economic fundamentals (Schnabel and Shin 2004). This paper provides a theoretical mechanism to understand how present bias, a main source of cognitive bias, contributes to financial fragility.

Empirical evidence has suggested that economic agents’ behavior is short-term oriented; thus, the discount functions are closer to hyperbolic than exponential (Ainslie 1992; Loewenstein and Prelec 1992). There are numerous applications of hyperbolic discounting in the economic literature. For example, Strotz (1956), Phelps and Pollak (1968) and Laibson (1996) were the first to apply the theory of hyperbolic discounting to households’ consumption-saving decision problems. Laibson (1997) further showed that consumers with hyperbolic discounting preferences face undersaving problems and demonstrated how financial innovation, which deprives consumers of self-commitment tools, paradoxically decreases their welfare. Since Laibson’s (1997) work, hyperbolic discounting time preferences have been used to explore a wide range of issues including consumption-savings choices, economic growth, financial investment, corporate investment, social security, human capital investment, and durable goods investment. However, to the best of my knowledge, no research has studied how a demand deposit system operates under hyperbolic discounting.

Based on field experiments in real banking sectors, the empirical evidence indicates that consumer preferences when they deposit and when they withdraw are time inconsistent. See Eckel, Johnson, and Montmarquette (2005), Ashraf, Karlan and Yin (2006). In particular, consumers are more prone to myopic behavior when early withdrawal is available (see Jones and Mahajan 2015). Based on this evidence, this paper constructs a Diamond-Dybvig model under hyperbolic discounting and investigates how time-inconsistency affects banking contracts and financial fragility.

Demand deposit contracts provide liquidity insurance for investors and depositors facing idiosyncratic liquidity shocks. These contracts provide considerable
savings and investment opportunities for ordinary consumers as well as large corporations. However, financial intermediaries providing demand deposit services are inherently unstable because of possible panic-based bank runs. Bank runs occur when investors and depositors rush to withdraw their deposits because they believe that others will also withdraw. Substantial theoretical research has explored panic-based bank runs. Diamond and Dybvig’s (1983) seminal work explained both the welfare gains of demand-deposit contracts and the risk of bank runs as a result of coordination failures. More recently, Peck and Shell (2003) introduced an endogenous bank run model showing that with a small probability of bank runs, banks could still provide a demand deposit contract that results in a higher level of welfare than autarkic allocation.

The main contribution of this paper lies in the combination of hyperbolic discounting preferences and demand deposit contracts. We incorporate hyperbolic discounting time preferences into a simple two-depositor model by Peck and Shell (2003) and Shell and Zhang (2018). This paper shows that present bias presents additional obstacles for a bank’s optimal demand deposit contract design. When a bank designs a contract, it needs to consider consumers’ tendency to choose early withdrawals due to present bias. Specifically, present bias affects incentive compatibility constraints and bank-run conditions. Incentive compatibility constraints ensure that patient consumers do not withdraw early. A no-bank-run condition prevents patient consumers from withdrawing if they believe other consumers will withdraw. This paper shows that under hyperbolic discounting, the two conditions become tighter under present bias when banks design an optimal contract; thus, either the bank tolerates a bank run (giving up on satisfying the no-run condition), or the bank ends up providing a contract that results in lower welfare than autarky because of the enforced incentive compatibility constraint.

This paper shows that the possibility of bank runs is derived from time inconsistency (because of present bias) but not from the slope of a patient depositor’s indifference curve. Standard Diamond–Dybvig studies assume that patient depositors equally value consumption in period 1 and 2, but Wallace (1988) extended the Diamond–Dybvig model by generalizing the patient depositor’s preferences (i.e., the slope of indifference curves). A present bias can make consumers’ deposit preferences differ from their withdrawal preferences, which can make it difficult for the bank to design the optimal contract. However, as long as the consumer is rationally time-consistent, deposit and withdrawal preferences are always ordinal equivalent, even under Wallace’s general utility functions. Thus, this paper shows that the patient consumer’s preference alone cannot be the source of bank runs.
We use the simple version of the endogenous bank-run model that was modified and proposed by Peck and Shell (2003) and developed by Shell and Zhang (2018) and Peck and Setayesh (2019). The Diamond and Dybvig (1983) model describes a post-deposit game that presumes that consumers have already deposited into a bank. Most of the research after Diamond and Dybvig (1983) described such post-deposit games.\(^1\) The few papers that modelled the pre-deposit game include Peck and Shell (2003), Ennis and Keister (2016), Shell and Zhang (2018) and Peck and Setayesh (2019).\(^2\)\(^,\)\(^3\) In the pre-deposit game, consumers decide to use the banking service or to stay in autarky. Peck and Shell (2003) first suggested a concrete example in which a bank run can occur in equilibrium. In their model, the bank run is triggered by sunspots (i.e., extrinsic uncertainty). Shell and Zhang (2018) later derived certain conditions in which the bank would tolerate a bank run in equilibrium. One of the main differences between their model and this paper is that they allowed heterogeneous utility functions of patient and impatient consumers, while we do not.\(^4\)

We assume that the bank is sophisticated in the sense that they design the banking contract by considering consumers’ future myopic decisions.\(^5\) Whether the consumers are sophisticated or naive does not affect ex-ante welfare through the banking service, because a sophisticated bank already considers consumers’ myopic behavior in the contract. However, it can affect welfare under autarky, showing that the misestimated ex-ante autarky welfare under naivety is higher than the correct welfare under sophistication.

Finally, this paper shows that present bias, in general, decreases consumers’ welfare. To measure welfare, we use normative preferences, which are defined based on exponential discounting and are commonly used in the hyperbolic discounting

\(^1\)There are two main differences between post- and pre-deposit games. First, in a post-deposit game, the welfare under autarky is not considered. However, in the pre-deposit game, if only the welfare under autarky is strictly lower than under the banking service, the consumer will choose to deposit. Second, in the pre-deposit game, sunspots (extrinsic uncertainty) can be used as a “random device” to choose the two equilibria of the post-deposit game. As shown in Peck and Shell (2003), multiple equilibria, including a run equilibrium in the posit-deposit game, are necessary for a run equilibrium in the pre-deposit game.

\(^2\)Peck and Setayesh (2019) created a bank model where consumers can choose how much to deposit. They showed that lower deposit levels make the banking system more fragile because a lower deposit level tempts a patient consumer to withdraw early and join a run.

\(^3\)As shown in Peck and Setayesh (2019), in a typical economy with identical utility functions, the incentive compatibility constraint does not bind at the optimal banking contract.

\(^4\)Specifically, Peck and Shell (2003) showed that when the impatient consumer’s marginal utility of consumption is greater than that of the patient consumer, there can be an endogenous bank run. The high marginal utility of consumption of the impatient consumer induces the early withdrawal amount to be large, which can be a main source for the bank run.

\(^5\)The naive case is also discussed in Appendix B.
literature (see O’Donoghue and Rabin 1999, 2005). When the degree of present bias is small, there is no change in the banking contract, resulting in no change in welfare. However, when present bias is above a certain range, incentive constraints or run conditions can affect the optimal contract in such a way to decrease the utility gain from the banking contract. If the present bias is considerably larger, the bank cannot provide a demand contract that increases welfare to a level higher than the autarkic one.

The remainder of the paper is organized as follows. In Section 2, we introduce a three-period bank-run model with present bias. Section 3 investigates how the inconsistency of time preferences introduces difficulties for the bank in designing a demand deposit contract. Section 4 investigates how present bias affects the possibility of bank runs and the welfare of banking services. Section 5 extends the model to include many consumers and an endogenous level of deposits and shows that hyperbolic discounting, in general, makes the banking system more fragile. Section 6 concludes. All proofs are in the appendices.

2. The Model

2.1. Patient and impatient consumers

Our basic framework is the models of Peck and Shell (2003) and Shell and Zhang (2018), which incorporate extrinsic uncertainty (i.e., sunspots) into the Diamond and Dybvig (1983) bank-run model. There are two consumers and three periods: 0, 1, and 2. In period 0, each consumer is endowed with $y$ units of the consumption good but there is no endowment in periods 1 and 2.

In period 1, each consumer becomes either impatient with probability $\pi$ or patient with probability $1 - \pi$. The two types of consumers are uncorrelated and have private information. Impatient consumers derive utility only from period-1 consumption. However, patient consumers derive utility from either period-1 or period-2 consumption. In the Diamond-Dybvig bank-run model, consumers’ patience in consumption can be interpreted in two ways. First, the patient consumer derives utility only from period-2 consumption, but she has storage technology to move resources from period 1 to period 2 without cost. Thus, even though the patient consumer withdraws deposits in period 1, the withdrawn consumption good can be transferred into period 2 using the storage technology. The second inter-

\[\text{In Section 5, we extend the model including the case of many consumers and an endogenous level of deposits.}\]
pretation is that the patient consumer is indifferent between consuming in period 1 or period 2 and there is no storage technology. Specifically, the utility gain from period-1 consumption is the same as the utility gain from period-2 consumption. In the Diamond-Dybvig model, both interpretations result in the same equilibrium outcome. In this paper, we adopt the second assumption to incorporate hyperbolic discounting into this model.

2.2. Technology and sequential service constraint

In this economy, there is technology in which investing 1 unit in period-0 consumption good yields $R > 1$ units if harvested in period 2 and yields one unit if harvested in period 1.\footnote{Even though the fundamental shocks are not a main concern in this paper, there is a way to incorporate the fundamental causes of runs, based on the work in Nikitin and Smith (2008), as we assume that the asset market return $\bar{R}$ is a random variable. If the information on the economy fundamentals is sufficiently negative, even patient consumers can choose to run for fear of a low value of return $\bar{R}$ that will be realized in the last period.} In period 0, the bank designs a deposit contract to maximize consumers’ ex-ante utility. The mechanism of withdrawal satisfies the sequential service constraint. Consumers who withdraw in period 1 are assumed to arrive at the bank in random order, and consumers do not know their positions in the bank queue. If more than one consumer chooses to withdraw, we assume that the positions in the queue are equally probable.\footnote{This basic model of this paper is from Peck and Shell (2003) that assumed that depositors do not know their position in the service queue. However, a different assumption commonly used in the literature (e.g., Green and Lin 2003) is that each depositor knows the clock time at which he/she arrives at the bank.}

In period 0, the bank chooses the optimal contract to maximize the consumer’s expected utility. Each consumer in period 0 chooses whether or not to deposit her entire endowment. When her expected utility under banking services is higher than that under autarky (i.e., an economy with no bank), she will choose to deposit. If she chooses to deposit, she will move on to the second stage in which she makes a decision on the timing of the withdrawal. In this sequential game, the optimal contract can be solved using backward induction.

If the two consumers choose to deposit, the bank invests the total deposit, $2y$, in technology. Let $c$ denote the period-1 withdrawal of consumption by the consumer who arrives first. Where one consumer withdraws in period 1 and the other withdraws in period 2, they will withdraw $c$ and $(2y - c)R$, respectively. If both consumers choose to withdraw in period 1, the second consumer who arrives late will get $(2y - c)$ from the bank. If both consumers choose to withdraw in period 2, each can withdraw $yR$. Each consumer withdrawing in period 2 receives the same
consumption due to expected utility maximization. See Table 1 for a summary of the withdrawal timing and amounts.

<table>
<thead>
<tr>
<th></th>
<th>Period 1 withdrawal</th>
<th>Period 2 withdrawal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both choose period 1</td>
<td>$c$, $2y - c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>first arrival</em></td>
<td><em>second arrival</em></td>
</tr>
<tr>
<td>Periods 1 and 2</td>
<td>$c$</td>
<td>$(2y - c)R$</td>
</tr>
<tr>
<td>Both choose period 2</td>
<td></td>
<td>$yR$, $yR$</td>
</tr>
<tr>
<td></td>
<td><em>first arrival</em></td>
<td><em>second arrival</em></td>
</tr>
</tbody>
</table>

Table 1. Timing of withdrawal and its amount

2.3. Hyperbolic discounting

We incorporate hyperbolic discounting into the bank-run model. According to hyperbolic discounting, valuations decrease relatively rapidly for earlier delay periods, but then decrease slower for longer delay periods. For example, Ainslie’s (1992) research showed that a substantial number of subjects reported that they would prefer $50 immediately rather than $100 in 6 months, but would not prefer $50 in 3 months rather than $100 in 9 months.

Following the assumption in Diamond-Dybvig’s bank-run model, the patient consumer in period 0 is indifferent between one unit of period-1 consumption and one unit of period-2 consumption. Applying the theory of hyperbolic discounting, in period 1, the patient consumer strongly prefers one unit of consumption now to one unit of consumption in the next period. Thus, we introduce the present-biased parameter $\theta < 1$ such that in period 1, the patient consumer is indifferent between $\theta$ units of period-1 consumption and one unit of period-2 consumption. If the consumer is present biased, $\theta$ must be less than 1. The impatient consumers’ discounting rate between period 1 and period 2 is infinite, so applying hyperbolic discounting to impatient consumers’ preferences does not alter their decisions.

Let $c^1$ denote consumption received in period 1 and let $c^2$ denote consumption received in period 2. The consumers are identical ex-ante in period 0. Each agent has a state-dependent utility function with private information. With present-biased parameter $\theta$, the consumer’s state dependent utility function in period 0 is
where $\delta \in (1/R, 1)$ is the time-consistent discounting factor of the patient consumer between period 1 and period 2.\footnote{Wallace (1988) incorporated the discounting factor ($\delta$) into the Diamond-Dybvig bank-run model. This paper shows that the value of $\delta$ does not affect the possibility of bank runs so the only source of financial fragility is present bias (i.e., $\theta < 1$) in this model.} The impatient consumer can realize a utility gain only from period-1 consumption ($c^1$), while the patient consumer can gain utility either from period-1 and period-2 consumption. In the model, it is assumed that the consumption good is indivisible across time, so it is not possible for both $c^1$ and $c^2$ to be positive in Eq. (1).

In period 1, the impatient person obtains a utility of $u(c^1)$ from period-1 consumption but obtains zero utility from period-2 consumption. However, the patient consumer in period 1 obtains a utility gain of $u(c^1)$ from period-1 consumption but obtains a utility gain of $u(\theta \delta c^2)$ from period-2 consumption. For example, for $\theta = 0.5$ and $\delta = 1$, the consumer is indifferent between consuming one unit immediately or consuming two units in the next period. Therefore, in period 1, the consumer’s state-dependent utility should be

\[
\begin{align*}
\begin{cases}
  u(c^1) & \text{if impatient} \\
u(\theta c^1 + \theta c^2) & \text{if patient}
\end{cases}
\end{align*}
\]

From Eqs. (1) and (2), we know that the deviation of $\theta$ from 1 makes the withdrawing utility of Eq. (1) to be ordinally different from the depositing utility of Eq. (2). However, as long as $\theta = 1$, for any value of $\delta \in (0,1)$ the period-0 and period-1 utilities are ordinally equivalent. We assume that $u(c)$ is Constant-Relative-Risk-Aversion (CRRA) such as

\[
u(c) = \frac{c^{1-\sigma}}{1-\sigma},\]

where $\sigma > 1$ is the coefficient of relative risk aversion. Because we assume that $u(c)$ is CRRA, which is homothetic, the existence of $\theta$ does not affect the period-0 optimization problem. However, the present-biased parameter ($\theta$) affects period-1 preferences.

In the traditional consumption-savings model (including the macroeconomics representative-agent model), the discounting factor is applied to period-utility rather than to consumption. Therefore, in the $\beta$-$\delta$ model (i.e., a Laibson-style hyperbolic
discounting model), both time-inconsistent and time-consistent discounting factors $\beta$ and $\delta$ are applied to period-utility. However, the Diamond-Dybvig model assumes that the ordinal utility between $c_1$ and $c_2$ is linear and $u(c)$ is a cardinal utility. Wallace (1988) generalized the preference in the Diamond and Dybvig model and constructed a model where the discounting factor is applied to consumption. Following Wallace (1988)'s approach, the discounting factors $\delta$ and $\theta$ are applied directly to consumption rather than to utility function in this paper. For a detailed discussion, see Appendix A.

3. First-best allocations and bank-run conditions

The bank needs to maximize period-0 ex-ante welfare by considering the incentive compatibility constraint and run condition. This section investigates an unconstrained optimal contract (i.e., the first-best allocation), incentive compatibility constraints, and run conditions, respectively, and shows that the present bias affects the three differently.

3.1. Ex-ante welfare function without a bank run

In period 0, the bank maximizes the ex-ante welfare function. Assuming that there is no bank run, the welfare function from hyperbolically-discounting utility of Eq. (1) is given by

$$\tilde{W}(c) = \pi^2 \left[ \frac{u(\theta c)}{2} + \frac{u(\theta(2y-c))}{2} \right] + 2\pi(1-\pi) \left[ \frac{u(\theta c)}{2} + \frac{u(\theta(2y-c))}{2} \right] + (1-\pi)^2 \delta \theta R$$

In Eq. (3), there are three terms. Each term represents the following: both consumers are impatient with probability $\pi^2$; one consumer is patient and the other is impatient with probability $2\pi(1-\pi)$; and both consumers are patient with probability $(1-\pi)^2$.\textsuperscript{10} In period 0, the present bias parameter $\theta$ is applied to both types of consumption in periods 1 and 2. This ex-ante welfare in Eq. (3) can be the actual welfare function only if the patient consumer(s) does not choose early withdrawal. Therefore, we need to consider the incentive compatibility to prevent the patient consumer from withdrawing in period 1, as introduced in the following sub-sections.\textsuperscript{10}

\textsuperscript{10}Because contract $c$ is greater than $y$, there is no possibility that the impatient consumer will choose a later withdrawal.
Denote $c^H$ as the unconstrained optimal contract (i.e., the first-best allocation) that maximizes the ex-ante welfare function in Eq. (3):

$$c^H = \arg \max_{c \in [0,2y]} \hat{W}(c).$$

(4)

Because $u(c)$ is homothetic, $c^H$ is not affected by $\theta$. From Eqs. (3) and (4), $c^H$ is

$$c^H = \frac{2y}{\left(\frac{\pi}{2-\pi} + \frac{2(1-\pi)}{(2-\pi)(\delta R)^{\frac{1}{\sigma}}-\pi}\right)^{1/\sigma} + 1}. \quad (5)$$

As shown in Eq. (5), the unconstrained optimal contract $c^H$ is not affected by the present bias parameter $\theta$. However, as shown in the following two subsections, present bias affects the conditions for the run condition and incentive compatibility constraint. In Section 4, we show that the unconstrained optimal contract ($c^H$) is the same as the constrained optimal contract if there is no present bias (or $\theta = 1$). However, they are not necessarily the same with present bias, which implies that there could be a possibility of bank-run equilibrium.

### 3.2. Bank-run conditions

A bank run can occur in equilibrium if a patient consumer prefers to choose period 1 when the other consumer also chooses period 1. Thus, the necessary condition for the existence of a run equilibrium from hyperbolically-discounting utility functions of Eqs. (1-2) is

$$\frac{u(c)}{2} + \frac{u(2y-c)}{2} \geq u(\theta \delta (2y-c)R). \quad (6)$$

As for the equality of Eq. (6), the left side represents the patient consumer’s welfare under a bank run. If a run occurs, the period-1 withdrawal can be either the first or the second in the queue with the same probability (i.e., 1/2), so the expected utility should be $u(c)/2 + u(2y-c)/2$. The right side of Eq. (6) represents the patient consumer’s utility when she chooses period 2 but the other consumer chooses period 1. If the patient consumer decides to consume in period 2, the present bias parameter $\theta$ is applied, so the utility should be $u(\theta \delta (2y-c)R)$.

From Eq. (6), we can show that a run equilibrium exists if contract $c$ satisfies the following inequality:

$$c > c^R = 2y / \left[ \left( \frac{2}{(\theta \delta R)^{\frac{1}{(\sigma-1)}}} - 1 \right)^{1/(\sigma-1)} + 1 \right]. \quad (7)$$
Inequality in Eq. (7) indicates that if contract \( c \) is large, there would be a run equilibrium. As shown in Eq. (7), threshold \( c^R \) is decreasing in \( \theta \), which implies that present bias increases the possibility of a bank-run in equilibrium.

**Lemma 1** As the degree of present bias increases (i.e., as \( \theta \) decreases), \( c^R \) decreases, so the range of contract \( c \) with no-run condition (i.e., \( c \in (y, c^R) \)) shrinks for any value of \( \delta \in (0, 1] \)

**Proof:** Directly from Eq. (7).

Lemma 1 indicates that present bias can create a run equilibrium. This is because the patient consumer is more willing to choose early withdrawal under higher present bias.

The run condition in Eq. (6) presumes that the bank is sophisticated in the sense that it knows the consumer’s future preferences. If the bank is naive, in which case it believes that there would be no present bias in the future, it is trivial that the bank can face a bank run due to a contract design with the wrong information. We discuss this issue in Appendix C.

### 3.3. The incentive compatibility constraint

The deposit contract should induce patient consumers to choose period 2. Otherwise, the banking contract would be worse than autarky. Therefore, we have the following incentive compatibility constraint:

\[
\pi \left[ \frac{u(c)}{2} + \frac{u(2y - c)}{2} \right] + (1 - \pi)u(c) \leq \pi u(\theta \delta(2y - c)R) + (1 - \pi)u(\theta \delta yR). \quad (8)
\]

In Eq. (8), the left side represents the patient’s welfare when she chooses a period-1 withdrawal. With probability \( \pi \), the other consumer is impatient, in which case both consumers choose to withdraw in period 1. Then, the period-1 withdrawal can be either the first or the second in the queues with the same probability (i.e., 1/2), so the expected utility should be \( u(c)/2 + u(2y - c)/2 \). With probability \( (1 - \pi) \), the other consumer is patient, and thus she can withdraw \( c \).

In Eq. (8), the right side represents the patient consumer’s welfare where she chooses to withdraw in period 2. If the other consumer is impatient with probability \( \pi \), she can withdraw \( (2y - c)R \). If the other consumer is patient and chooses to withdraw in period 2, she will receive half of the bank’s return on the investment, \( yR \). Each consumer withdrawing in period 2 receives the same consumption due to expected utility maximization.
Denote $c^I$ as the level of $c$ such that Eq. (8) holds as equality. Eq. (8) implies that there is a certain value of $c^I \in (y, Ry)$ such that the optimal contract, $c$, satisfies the incentive compatibility constraint if and only if $c \leq c^I$.

**Lemma 2**  As the degree of present bias increases (i.e., as $\theta$ decreases), the range of contract $c$ where the incentive compatibility constraint is satisfied (i.e., $c \in (y, c^I)$) shrinks for any value of $\delta \in (0, 1]$

**Proof:** See Appendix C.

Lemma 2 indicates that present bias tightens the incentive compatibility constraints. This is because the patient consumer is more willing to choose early withdrawal under a higher degree of present bias.

Lemmas 1 and 2 show that the more present biased the consumers are, the lower the values of both $c^R$ and $c^I$. A lower value of $c^R$ can result in a bank run. A lower value of $c^I$ results in a decrease in the ex-ante welfare because the optimal contract $c$ cannot be higher than $c^I$. As shown in the following section, when $c^R < c^I$, the optimal contract $c$ is a value between $c^R$ and $c^I$ and a bank run in equilibrium can occur.

As demonstrated in Shell and Zhang (2018), we show that if $\sigma < (1, 2)$, $c^R < c^I$. This result holds for any value of $\theta \in (0, 1]$ as shown in the following lemma. Therefore, if $\sigma > 2$, there is no run in equilibrium. This result holds for any value of $\theta \in (0, 1]$.

**Lemma 3**  If $\sigma \in (1, 2)$, $c^R < c^I$ for any value of $\theta \in (0, 1]$ and $\delta \in (0, 1]$. If $\sigma > 2$, $c^R > c^I$ for any value of $\theta \in (0, 1]$ and $\delta \in (0, 1]$.

**Proof:** See Appendix D.

The optimal contract must satisfy the incentive compatibility constraint; otherwise, the demand contract cannot provide a higher welfare than autarky. Therefore, when $c^R > c^I$ (i.e., $\sigma > 2$ from Lemma 3), the optimal contract $c$ is lower than both $c^I$, which implies that it is also lower than $c^R$. In this case, there is no bank run in equilibrium but the welfare of the bank contract will be lower because of the tightened incentive compatibility constraint, as shown in the next section.

This section shows that the unconstrained optimal contract is not affected by present bias. However, the run condition and incentive compatibility are affected by present bias. In the next section, we investigate how present bias affects the optimal contract and the possibility of a bank run in equilibrium.
4. The pre-deposit game

In the pre-deposit game, there are three cases of equilibria. The first case is that the consumers choose autarky over the banking service when the banking service cannot provide higher welfare over autarky. The second case is that the consumer chooses the banking service even though she knows that the bank would tolerate small-probability runs. As Peck and Shell (2003) showed, there can be a threshold probability $s^0 \in (0, 1)$ such that if the sunspot run probability $s$ is larger than $s^0$, the optimal contract is the best run-proof contract. However, if the sunspot run probability $s$ is smaller than $s^0$, the optimal contract tolerates bank runs. The third case is that the bank does not tolerate runs.

Our main result is that present bias can move Case 3 (non-run equilibrium) to Case 1 (autarky) or Case 2 (run equilibrium) in the pre-deposit game. This means that present bias makes the financial market either useless (Case 1) or fragile (Case 2). As shown in Figure 1, as the degree of present bias increases, a run equilibrium can be created or autarky can be chosen in equilibrium.

4.1. Welfare under autarky

The pre-deposit game considers consumers’ choices between autarky and banking services. Under autarky, the individual will have direct access to the technology. If the consumer is patient, she will consume $yR$ in period 2. If the consumer is impatient, she will consume $y$ in period 1. Under the assumption that consumers are
sophisticated, her expected utility under autarky ($W^A$) is

$$W^A = \begin{cases} 
\pi u(\theta y) + (1 - \pi)u(\theta \delta y R) & \text{if } \theta \delta R \geq 1, \\
\pi u(\theta y) + (1 - \pi)u(\theta y) = u(\theta y) & \text{if } \theta \delta R < 1.
\end{cases}$$

If $\theta \delta R < 1$, the consumer knows that even if she turns out to be a patient consumer in period 1, she will choose to consume in period 1 because her utility by consuming in period 1, $u(y)$ is greater than her utility by consuming in period 2, $u(\theta y R)$. If the consumer is naive, regardless of her present-bias parameter, her estimated utility is

$$W^A = \pi u(\theta y) + (1 - \pi)u(\theta \delta y R) \tag{10}$$

In the next subsection, we investigate the welfare function under banking services and compare it with that of autarky.

### 4.2. Extrinsic Uncertainty in Sunspots

In the pre-deposit game, the bank announces its optimal contract $c$ at the beginning of period 0. Then, the consumer decides whether or not to deposit in period 0. At the beginning of period 1, each consumer learns her type after observing a sunspot signal $\rho$. Sunspots do not affect preferences, endowments, technology, or the probability of being patient. However, the consumer’s decision on withdrawal timing can depend on the sunspot signal as well as her consumer type. Without loss of generality, we assume that the sunspot variable $\rho$ is uniformly distributed on $[0, 1]$. We also assume that the bank cannot choose a withdrawal schedule that directly depends on $\rho$.

Consumers withdraw as follows. For $\rho < s$, all consumers choose period 1 if the bank-run condition is satisfied. If the sunspot signal is not observed (i.e., $\rho > s$), all patient consumers will withdraw in period 2 regardless of whether the post-deposit subgame has multiple equilibria or single equilibrium. Because $\rho$ is uniformly distributed on $[0, 1]$, the value of $s$ can be interpreted as a bank-run probability. However, observing the sunspot signals does not necessarily result in a bank run in the pre-deposit game. All patient and impatient consumers will choose to withdraw in period 1 with sunspot signals only when the run condition is satisfied (i.e., $c > c^R$).
4.3. Welfare in a pre-deposit game

Let $W_{\text{run}}$ denote the depositor’s period-0 expected utility in a run equilibrium when the inequality in Eq. (6) is satisfied. It is given by

$$W_{\text{run}}(c) = \frac{u(\theta c)}{2} + \frac{u(\theta (2y - c))}{2}. \tag{11}$$

If contract $c$ is smaller than $c^R$, a run equilibrium cannot exist and the ex-ante expected utility would be the same as the unconstrained welfare, $\hat{W}(c)$. However, if $c^R < c$, both run and nonrun equilibria exist in the post-deposit game. Therefore, the period-0 ex-ante expected utility $W(c; s)$, given the run probability $s$, should be

$$W(c; s) = \begin{cases} \hat{W}(c) & \text{if } c \leq c^R \\ (1 - s)\hat{W}(c) + sW_{\text{run}}(c) & \text{if } c^R < c \leq c^I \end{cases} \tag{12}$$

Let $c^*(s)$ denote the optimal contract which maximizes the depositor’s ex-ante utility in the pre-deposit game, given the run probability $s$. Then, we have

$$c^*(s) = \arg \max_{c \in [0,c^I]} W(c; s). \tag{13}$$

We first show that if consumers are time consistent (or $\theta = 1$), the optimal contract is run proof for any value of $s \in [0,1]$, as shown in the following proposition:

**Proposition 1** If the consumers are time consistent (or $\theta = 1$), the optimal contract $c^*(s)$ is run proof for any value of $s \in [0,1]$ and thus the consumers will always choose to deposit over maintaining autarky.

**Proof:** See Appendix E.

Proposition 1 shows that there will be no bank-run equilibrium in the pre-deposit game if the consumer is time-consistent, i.e., $\theta = 1$.

The main contribution of this paper is to show that present bias could be a source of financial fragility. The banking contract is announced earlier in period 0 before depositors make a decision on an early withdrawal later in period 1. When the bank designs the optimal contract in period 0, it should consider the consumers’ future myopic preferences to prevent this myopic behavior that could result in a bank run. Specifically, two types of conditions are affected by present bias. One condition is the incentive compatibility constraint to induce patient consumers to choose later withdrawals. If the contract does not satisfy the incentive compatibility constraint, the banking contract results in lower welfare than autarky. With present bias, the
patient consumer would be more willing to choose early withdrawal, as shown in Lemma 2, so the optimal contract $c^*$ would be designed in a more restrictive way. With high present bias, it is possible that the incentive compatibility constraint would rule out any contract $c^*$ that is greater than $y$, which implies that the bank cannot provide any risk-sharing opportunity.

The present bias also has a direct impact on the run condition as shown in Lemma 1. With the same logic for the incentive compatibility constraint, present bias increases the possibility of a run (or decreases $c^R$). Thus, present bias increases the chance that the bank would tolerate a bank run in the pre-deposit game. The following proposition summarizes the impact of present bias on the banking economy:

**Proposition 2** If $\sigma \in (1, 2)$, there exist $\theta_L, \theta_U \in (0, 1)$ with $\theta_L < \theta_U$ such that for any $\theta \in (\theta_L, \theta_U)$, the optimal contract $c^*(s)$ tolerates runs for a small value of $s$; and for any $\theta \in (0, \theta_L)$, $c^*(s)$ cannot result in higher welfare than autarky. If $\sigma \in (2, \infty)$, there exists $\theta_B \in (0, 1)$ such that for any $\theta < \theta_B$, the banking contract cannot provide higher welfare than autarky.

**Proof:** See Appendix F.

<table>
<thead>
<tr>
<th>$\sigma \in (1, 2)$</th>
<th>High present bias or $\theta &lt; \theta_L$ or $\theta &lt; \theta_B$</th>
<th>Medium range of $\theta$ or $\theta \in (\theta_L, \theta_U)$</th>
<th>No present bias $\theta = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^R &lt; y$</td>
<td>$\begin{align*} y &amp;&lt; c^R &lt; c^I &lt; c^H \ y &amp;&lt; c^R &lt; c^H &lt; c^I \end{align*}$</td>
<td>$\begin{align*} y &amp;&lt; c^H &lt; c^R &lt; c^I \end{align*}$</td>
<td>A bank run could exist</td>
</tr>
<tr>
<td>$\sigma \in (2, \infty)$</td>
<td>$c^I &lt; y$</td>
<td>$\begin{align*} y &amp;&lt; c^I &lt; c^R &lt; c^H \ y &amp;&lt; c^H &lt; c^R &lt; c^I \end{align*}$</td>
<td>$\begin{align*} y &amp;&lt; c^H &lt; c^I &lt; c^R \end{align*}$</td>
</tr>
<tr>
<td>Autarky is chosen</td>
<td>No bank run</td>
<td>No bank run</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Present bias and optimal deposit contracts

Proposition 2 indicates that for a lower value of $\theta$, it is possible that the bank would tolerate a bank run in equilibrium if $\sigma \in (1, 2)$. As shown in Lemma 3, for a medium range of risk aversion ($\sigma \in (1, 2)$), we have $c^R < c^I$ for any level of
present bias $\theta \in (0, 1]$. In this case, there is a run equilibrium for a certain range of present bias $\theta \in (\theta_L, \theta_U)$. For a higher level of risk aversion, we have $c^R > c^I$, so a run equilibrium cannot exist for any value of $\theta \in (0, 1]$. In both cases, for a lower value of present bias parameter $\theta$, a banking contract cannot provide higher welfare than autarky.\[12\] Table 2 summarizes the result in Proposition 2. The example in Appendix J shows how a run equilibrium occurs with present bias for the case of $\sigma \in (1, 2)$.

4.4. Consumers’ welfare with present bias

In this subsection, we investigate consumers’ welfare in a banking economy with present bias. With present-biased preferences, there is no single dominating welfare measure because there are multiple preferences one can use. Therefore, it is common to use a welfare function that is not affected by the present-bias parameter, commonly called normative utility or paternalistic utility (see O’Donoghue and Rabin 1999a, 2005)\[13\]. This normative utility is defined as letting $\theta = 1$ in period-0 intertemporal utility. This utility is used to assess policy implications in the literature on hyperbolic discounting preferences. In this setting, the normative preference should be

$$W^N = \pi u(c^1) + (1 - \pi)u(\delta c^2)$$

From the definition of the normative preferences in Eq. (14), we have the following proposition:

**Proposition 3** As $\theta$ decreases, consumers’ normative utility decreases (not necessarily strictly decreasing).

**Proof:** See Appendix G.

Proposition 3 shows that an increase in present bias has a negative impact on welfare in a banking economy. There are three ranges of $\theta$. First, with a very small

---

\[11\] The present bias parameter $\theta$ does not affect the inequality $c^R < c^I$ or $c^R < c^I$. Even though decreased $\theta$ decreases both $c^R$ and $c^I$, it cannot reverse the inequality under the homothetic preferences.

\[12\] Peck and Shell (2003) and Shell and Zhang (2018) focused on the case when a bank run exists, i.e., the case with a medium value of risk aversion (or $\sigma \in (1, 2)$).

value of $\theta$, as shown in area I in Figure 2, banking cannot provide higher welfare than autarky because the incentive compatibility strongly constrains the optimal banking contract. Put simply, the optimal contract $c^*$ cannot be higher than $y$; otherwise, incentive compatibility is violated. In the middle range of $\theta$, as shown in area II, banking can exist but incentive compatibility still affects the optimal contract. In this case, the bank may or may not tolerate runs. If $s$ is small and $c^R < c^I$, which is the case of the example in Appendix J, the bank tolerates runs. However, regardless of whether a run is possible or impossible, welfare decreases under present bias within this range because the incentive constraints make the optimal contract more restrictive. For a sufficiently high value of $\theta$, as shown in area III in Figure 2, incentive compatibility is not binding, so the welfare is not affected by the variation of $\theta$.

5. Extensions to the Peck-Setayesh (2019) model

The main model in this paper assumes that there are only two consumers, in which each decides whether or not to deposit. This section extends the model to many consumers in which they can endogenously choose the amount they wish to deposit, based on the work of Peck and Setayesh (2019).

This section assumes that there is a finite number, $N$, of consumers. The number of impatient consumers, denoted by $\alpha$, is a random variable with probability
distribution $f(\alpha)$. From Bayes’ rule, the distribution of $\alpha$, conditional on being patient, is given by

$$f_p(\alpha) = \frac{(1 - \alpha/N) f(\alpha)}{\sum_{0=a}^{N-1} (1 - a/N) f(a)}.$$  

At the beginning of period 0, the bank chooses the deposit level, $d$, and the amounts of period-1 and period-2 withdrawals.\textsuperscript{14} Withdrawals can be characterized by a pair of non-negative functions, $c_1(z,d)$ and $c_2(\alpha_1,d)$. For $z = 1, \ldots, N$, $c_1(z,d)$ is the withdrawal amount for the $z^{th}$ consumer to arrive in period 1 with the deposit level, $d$. For $\alpha = 0, \ldots, N - 1$, $c_2(\alpha_1,d)$ is the withdrawal amount in period 2 when the number of consumers withdrawing in period 1 is $\alpha_1$. Then, the bank’s resource constraints are

$$c_1(N,d) = dN - \sum_{z=1}^{N-1} c_1(z,d) \quad (15)$$

and

$$c_2(\alpha_1,d) = \left[ dN - \sum_{z=1}^{N-1} c_1(z,d) \right] R \quad (16).$$

An impatient consumer withdrawing $c_1$ in period 1 receives utility, $u(1 - d + c_1)$. A patient consumer withdrawing $c_t$ in period $t$ receives utility, $u((1 - d) R + c_t)$. For convenience, we use the notation from Peck and Setayesh (2019): $x_1(z,d) = 1 - d + c_1(z,d)$ and $x_2(\alpha_1,d) = (1 - d) R + c_2(\alpha_1,d)$. Then, assuming that there is no bank run, the welfare function from hyperbolically-discounting utility is given by

$$\bar{W}(c,d) = \sum_{\alpha=0}^{N-1} f(\alpha) \left[ \sum_{z=1}^{\alpha} u(\theta x_1(z,d)) + (N - \alpha) u(\theta \delta x_2(\alpha,d)) \right] + f(N) \left[ \sum_{z=1}^{N} u(\theta x_1(z,d)) \right]. \quad (17)$$

By the continuity property, we know that there exists a first-best solution $\{c_1^H(z,d), c_2^H(\alpha_1,d)\}$ from Eqs. (15-17). A bank run can occur in equilibrium if a patient consumer prefers to choose period 1 when the other consumer chooses period 1. Thus, the necessary condition for the existence of a run equilibrium from hyperbolically-discounting

\textsuperscript{14}Peck-Setayesh (2019) proves that in the endogenous deposit-level model, both the bank and consumers have an incentive to choose the maximum deposit level (i.e., $d = 1$). This is because a partial deposit ($d < 1$) makes both incentive constraints and bank-run conditions tighter while keeping the first-best allocations unchanged. Consequently, choosing partial deposits rather than full deposits results in a decrease in welfare. Therefore, Peck-Setayesh’s (2019) model is equivalent to Peck-Shell’s (2003) model in the setting of this paper. Peck-Setayesh (2019) show that partial deposit can exist in equilibrium only when the asset return outside the bank is higher than the return of the technology the bank accesses.
utility functions is
\[
\frac{1}{N} \sum_{z=1}^{N} u((1 - d)(R - 1) + x_1(z, d)) \geq u(\theta \delta x_2(N - 1, d)).
\] (18)

The incentive compatibility constraint for a patient consumer is given by
\[
\sum_{a=0}^{N-1} f_p(\alpha) \left[ \frac{1}{1+\alpha} \sum_{z=1}^{\alpha+1} u((1 - d)(R - 1) + x_1(z, d)) \right] \leq \sum_{a=0}^{N-1} f_p(\alpha) u(\theta \delta x_2(\alpha, d)).
\] (19)

From Eq. (17), we know that the first-best allocation is not affected by the present bias \( \theta \). However, both the incentive compatibility constraint and the bank-run condition are affected, as shown in Eqs. (18-19). Specifically, the two conditions in Eqs. (18-19) become tighter so it is difficult for the bank to design the optimal contract. The following lemma shows that the present bias \( \theta \) does not affect the first-best allocations but makes the two conditions tighter:

**Lemma 4** The first-best allocation \( \{c_1^H(z, d), c_2^H(\alpha_1, d)\} \) is not affected by \( \theta \). However, there exists \( \theta' \in (0, 1] \) such that for any value of \( \theta \leq \theta' \), the incentive compatibility in Eq. (18) constraint is violated or the bank run condition is satisfied at \( \{c_1^H(z, d), c_2^H(\alpha_1, d)\} \).

**Proof**: See Appendix H.

Lemma 4 shows that even in Peck-Setayesh’s (2019) model, in general, the existence of hyperbolic discounting can increase the possibility of bank runs. Unlike the model with two consumers, \( \theta' \) can have a value of one. This implies that even without present bias (\( \theta = 1 \)), a run equilibrium can exist, as shown in Peck and Setayesh (2019).

Now, we assume that the extrinsic uncertainty in sunspots can trigger a bank run in a pre-deposit game as described in the previous section. The following proposition shows that with the result of Lemma 1, in the pre-deposit game, the present bias can be a cause of financial fragility:

**Proposition 4** In a pre-deposit game, there exists \( \theta' \in (0, 1] \) such that for any value of \( \theta \leq \theta' \), the optimal contract tolerates runs for a small value of \( s \) or the bank cannot have a higher welfare than autarky.

**Proof**: See Appendix I.
Proposition 4 indicates that the value of $\theta'$ can be one, which means that even without present bias, the bank can tolerate bank runs in equilibrium. This is in contrast to the result of the two-consumer model in which the bank run cannot exist in equilibrium if $\theta = 1$. There is another difference between the extended model in this section with the two-consumer model; In the two-consumer model, Shell and Zhang (2019) showed that depending on the value of relative-risk-aversion ($\sigma$), whether the incentive compatibility constraint are the bank-run condition are binding. However, in the extended model of this section, we still do not know how the two constraints are determined by the relative-risk-aversion. Therefore, in Proposition 4, we could not clarify how the two cases of financial fragility (having a bank run in equilibrium or the bank market being useless) differ depending on the risk aversion value.

6. Summary and Discussion

This paper shows that present bias can result in the following three equilibrium cases in the pre-deposit game:

Case 1: Present bias can create a bank run in equilibrium. In this case, the incentive compatibility constraint may or may not be binding. However, in both cases, with a small probability, a bank run can occur in equilibrium.

Case 2. There can be a run equilibrium, which results in a strong negative impact on ex-ante welfare under the banking service. Then, it is possible that welfare under autarky is even higher than that under the banking service, so consumers have no incentive to use the banking service.

Case 3: The incentive compatibility constraint more strictly binds the contract such that the banking contract becomes less welfare-improving. In this case, it is possible that welfare under autarky is greater than that under the banking service, so consumers have no incentive to use the banking service.

These three cases presume that the degree of present bias is large enough to change the banking contract and welfare. In the model, if the degree of consumers’ present bias is small, it is possible that the optimal banking contract and ex-ante welfare are not affected by the present bias.

This paper also shows that bank runs due to consumers’ myopia causes a considerable welfare loss for consumers. Therefore, government policies can help improve welfare as it prevent bank runs. An efficient way to prevent bank runs is to insure patient depositors get the full withdrawals in period 2, as proposed by Diamond and Dybvig (1983). If the bank is not able to fulfill its obligations, the government
can pay the difference and a deposit insurance system can be financed by the bank or taxes. In the model in this paper, a simple deposit insurance system can help prevent a bank run, as explained by the bank run condition in Eq. (6). Under a deposit insurance system, the government can provide a certain amount of money, \( m \), under a run situation. Then, the bank run condition in Eq. (6) can be modified as

\[
\frac{u(c)}{2} + \frac{u(2y - c)}{2} \geq u(\theta \delta (2y - c) R + m).
\]

From Eq. (20), we know that there exists \( m > 0 \), which prevents a bank run (i.e., making the inequality in Eq (20) not satisfied). As the government promises to pay \( m \) under bank runs, it can effectively prevent a run. If the two consumers are impatient, they will choose period 1 and give up \( m \). For the other cases where one of the consumers is impatient, there will be no bank run and the government does not need to pay \( m \). For this simple model without fundamental shocks, the government would prevent a run by ensuring that patient consumers would receive the full value of their deposits.

**Appendices**

**A. Discounting factors and hyperbolic discounting**

The discounting factors in this paper are modelled differently compared to the traditional approaches of Phelps and Pollack (1968) and Laibson (1997), called the \( \beta\delta \) model. For those models, the present bias parameter (or hyperbolic discounting factor) is applied to utility function (or \( \theta u(c) \)). However, in our model, it is applied directly to consumption (or \( u(\theta c) \)). Our setting of the model is based on Wallace (1988) in which the consumer’s ex-ante utility function is defined as

\[
\pi u(c_1 + \delta_I c_2) + (1 - \pi) u(c_1 + \delta_P c_2),
\]

where \( \delta_I \) and \( \delta_P \) are the discounting factors of the impatient and patient consumers, respectively, and \( \delta_I < 1/R < \delta_P \). In the Diamond-Dybvig model, it is assumed that \( \delta_I = 0 \) and \( \delta_P = 1 \). Since \( c_1 \) and \( c_2 \) are perfect substitutes in Diamond-Dybvig (1983) and Wallace (1988), we should directly apply the discounting factors (\( \delta \) and \( \theta \)) to the consumption unit but not apply them to cardinal utility \( u(\cdot) \).
In contrast, Laibson (1997) incorporates the hyperbolic discounting model into the conventional consumption-savings model. Specifically, he explained that the hyperbolic discounting factor $\beta$ can explain the time-inconsistent myopic behavior in terms of the marginal rate of substitution. Specifically, Laibson (1997) write the following on page 451:

“The preferences ... are dynamically inconsistent, in the sense that preferences at date $t$ are inconsistent with preferences at date $t + 1$. To see this, note that the marginal rate of substitution between periods $t + 1$ and $t + 2$ from the perspective of the decision-maker at time $t$ is given by $u'(c_{t+1})/(\delta u'(c_{t+2}))$, which is not equal to the marginal rate of substitution between those same periods from the perspective of the decision-maker at $t + 1$: $u'(c_{t+1})/(\beta \delta u'(c_{t+2}))$.”

In the time-inconsistent model of this paper, it is also possible to show that there is inconsistency in the marginal rate of substitution, which is similar to Laibson’s approach. In our model, in period 0, the ex-ante utility function is

$$\pi_I u(c_1 + \delta_I c_2) + \pi_P u(c_1 + \delta_P c_2),$$

and in period 1, the ex-ante utility is

$$\pi_I u(c_1 + \theta \delta_I c_2) + \pi_P u(c_1 + \theta \delta_P c_2).$$

The patient consumer’s marginal rate of substitution between periods 1 and 2 from the perspective of the decision-maker at time 0 is

$$1/\delta_p. \quad (22)$$

The patient consumer’s the marginal rate of substitution between periods 1 and 2 from the perspective of the decision-maker at time 1 is

$$1/(\theta \delta_P), \quad (23)$$

which is higher then $\delta_p$. If $\delta_p = \delta$, $\theta = \beta$, and the utility function in Laibson, $u(\cdot)$, is linear; the marginal rates of substitution in Eqs. (22-23) are exactly the same as $u'(c_{t+1})/(\delta u'(c_{t+2}))$ and $u'(c_{t+1})/(\beta \delta u'(c_{t+2}))$ in Laibson (1997).

Nevertheless, if the range of the utility functions is the set of positive numbers, the two settings do not result in different outcomes for any type of utility function $u(\cdot)$. However, if the range is negative, the lower value of $\theta$ in $\theta u(c)$ means a higher value of utility, which implies that the consumer has future bias rather than present
bias. For example, the utility function is \(-c^{-1}\) and the hyperbolic discounting factor is \(\beta = 0.5\), and the long-term discounting factor is \(\delta = 1\). We assume that the consumer consumes \((c_1, c_2)\) where \(c_1\) and \(c_2\) represent the consumption in periods 1 and 2, respectively. In this case, the consumer is indifferent between \((1, \frac{1}{2})\) and \((\frac{1}{2}, 1)\) in period 0. However, based on the hyperbolic discounting factor with \(\beta = 0.5\), the consumer strictly prefers \((\frac{1}{2}, 1)\) to \((1, \frac{1}{2})\) in period 1 because we have

\[
-c_1^{-1} - \beta c_2^{-1} = \begin{cases} 
-2 + 0.5 \times (-1) = -2.5 & \text{where } (c_1, c_2) = (1/2, 1) \\
-1 + 0.5 \times (-2) = -2 & \text{where } (c_1, c_2) = (1, 1/2) 
\end{cases} 
\]  
(24)

The result in Eq. (24) implies that the consumer is future biased rather than present biased. This is opposite to the concept of hyperbolic discounting.

The hyperbolic discounting factor \(\beta\) in the conventional \(\beta-\delta\) model indicates that the marginal utility is hyperbolically discounted but is not the utility value of consumption. That is, the marginal utility of period 3 relative to period 2 is \(\beta(<1)\) if it is evaluated in period 1, but it is one if it is evaluated in period 0. Because most hyperbolic models analyze the marginal utility in the utility maximization problem, the hyperbolic discounting parameter in these models could be applied to the utility function with a negative range.

To make both the utility value and marginal utility hyperbolically discounted, we make a positive linear transformation on the utility for its domain to be positive. For example, we define the utility function as

\[u(c) = -c^{-1} + 10, \text{ where } c > 1/10.\]

Thus, both marginal utility and utility are hyperbolically discounted in the \(\beta-\delta\) model.

**B. The case in which the bank is naive**

If the bank is naive (i.e., the bank does not expect consumers’ future myopic behavior),
\(^{15}\) it will face a high risk of bank runs or a violation of an incentive compatibility constraint. Specifically, a naive bank wrongly assumes that \(\theta = 1\).

Denote \(c^{I}_N\) and \(c^{R}_N\) as the naive bank’s estimated value of \(c^{I}\) and \(c^{R}\). Then, from

\(^{15}\)The sophistication and naivety of a bank were introduced by Ennis and Keister (2003) but in a different context. In their model, a bank is considered sophisticated if it realizes the endogeneity of the likelihood of runs. Otherwise, the bank is naive.
Lemmas 1 and 2, which show that \( c^R \) and \( c^I \) increase with decreased \( \theta \), we know
\[
 c^I_N < c^I \quad \text{and} \quad c^R_N < c^R. \tag{25}
\]

From Eq. (5), we know that contract \( c^* \) could violate the incentive constraint. Then, the welfare from the bank is lower than autarkic allocations. Therefore, the welfare with a naive bank is lower than that of a sophisticated bank. Specifically, where \( c^I < c^H \), a naive bank will choose \( c^H \) (rather than \( c^I \)) as the contract because it believes that \( c^H < c^I_N \). In this case, the incentive compatibility constraint is violated so patient consumers would choose early withdrawal, which would result in a lower welfare than autarky allocation.

If \( c^I < c^H \) and the bank is naive, period-0 welfare is the same as that of the bank-run case, as shown in Eq. (11):
\[
 W^{\text{run}}(c^H) = \frac{u(\theta c^H)}{2} + \frac{u(\theta (2y - c^H))}{2}. \tag{26}
\]
which is lower than the welfare under autarky.

C. Proof of Lemma 2

From Eq. (8), we know that \( c^I \) satisfies the following equation:
\[
 \pi \left[ \frac{u(c^I)}{2} + \frac{u(2y - c^I)}{2} \right] + (1 - \pi)u(c^I) = \pi u \left( \theta \delta (2y - c^I) R \right) + (1 - \pi) u \left( \theta \delta y R \right). \tag{27}
\]

Implicitly differentiating Eq. (27) with respect to \( \theta \), we have
\[
 \frac{\pi}{2} u'(c^I) dc^I - \frac{\pi}{2} u'(2y - c^I) dc^I + (1 - \pi) u'(c^I) dc^I = \pi u' \left( \theta \delta (2y - c^I) R \right) \delta (2y - c^I) R d\theta - \pi u' \left( \theta \delta (2y - c^I) R \right) \theta \delta R dc^I
\]
which is, in turn, equivalent to
\[
 \frac{d c^I}{d\theta} = \frac{\pi u' \left( \theta \delta (2y - c^I) R \right) \delta (2y - c^I) R}{\frac{\pi}{2} u'(c^I) - \frac{\pi}{2} u'(2y - c^I) + (1 - \pi) u'(c^I) + \pi u' \left( \theta \delta (2y - c^I) R \right) \theta \delta R} > 0,
\]
which implies that \( c^I \) is strictly increasing in \( \theta \).
D. Proof of Lemma 3

As in Eq. (7), we have
\[ c^R = 2y / \left[ \left( \frac{2}{(\theta \delta R)^{\sigma-1}} - 1 \right)^{1/(\sigma-1)} + 1 \right] . \]  \hspace{1cm} (28)

Because the left side of Eq. (27) is increasing in \( c^I \) while the right side is decreasing in \( c^I \), the condition that \( c^R < c^I \) can be derived by replacing \( c^I \) with \( c^R \) in Eq. (27):
\[ \pi \left[ \frac{u(c^R)}{2} + \frac{u(2y - c^R)}{2} \right] + (1 - \pi)u(c^R) < \pi u \left( \theta \delta (2y - c^R)R \right) + (1 - \pi)u \left( \theta \delta y R \right) . \]  \hspace{1cm} (29)

The inequality in Eq. (29) is equivalent to
\[ 2/ (\theta \delta R) < \left( 1/ (\theta \delta R)^{\sigma-1} - 1 \right)^{1/(\sigma-1)} . \]  \hspace{1cm} (30)

Because \( \left( 1/ (\theta \delta R)^{\sigma-1} - 1 \right)^{1/(\sigma-1)} \) is decreasing in \( \sigma \), the inequality in Eq. (30) is equivalent to \( \sigma < 2 \).

E. Proof of Proposition 1

For proof of Proposition 1, we need to show that if \( \theta = 1 \), (1) there would be no bank-run equilibrium (i.e., \( c^H < c^R \)) and (2) welfare under the banking contract is higher than under autarky i.e., \( (W(c^*) > W^A) \). The two statements will be proven in the following two lemmas:

Lemma 5 We have \( c^H < c^R \) if \( \theta = 1 \).

Proof: Lemma 5 indicates that the unconstrained optimal contract \( c^H \) is smaller than \( c^R \) if \( \theta = 1 \). This implies that there would be no bank run in equilibrium regardless of the value of \( c^I \). From Eqs. (5) and (7), we know that \( c^H < c^R \) implies that
\[ \frac{2y}{\pi} + \frac{2(1-\pi)}{(2-\pi)(\delta R)^{\sigma-1}} \frac{1}{\sigma} + 1 \leq \frac{2y}{\left( \frac{2}{(\delta R)^{\sigma-1}} - 1 \right)^{1/(\sigma-1)} + 1} . \]
which is, in turn, equivalent to

$$
\left( \frac{1-\pi}{2-\pi} \left( \frac{\pi}{1-\pi} + \frac{2}{(\delta R)^{\sigma-1}} \right) \right)^{1/\sigma} > \left( \frac{2}{(\delta R)^{\sigma-1}} - 1 \right)^{1/(\sigma-1)}.
$$

(31)

Since $(\delta R) > 1$, we have $\frac{1-\pi}{2-\pi} \left( \frac{\pi}{1-\pi} + \frac{2}{(\delta R)^{\sigma-1}} \right) > 1$ and $\frac{2}{(\delta R)^{\sigma-1}} - 1 < 1$. Therefore, for any values of $\sigma > 1$, inequality of Eq. (31) is satisfied. **End of Proof.**

**Lemma 6** We have $\hat{W}(c^*) > W^A$ if $\theta = 1$.

**Proof:** Because there is no bank-run equilibrium, as shown in Lemma 5, and we have $W^A = \hat{W}(y)$, the sufficient condition for $\hat{W}(c^*) > W^A$ is $c^* > y$. Because $c^* = \min(c^I, c^H)$, we can prove Lemma 6 by showing that $c^I > y$ and $c^H > y$.

The following is the proof for $c^H > y$. As in Eq. (5), we have

$$
c^H = \frac{2y}{\left( \frac{\pi}{\pi-\pi} + \frac{2(1-\pi)}{(2-\pi)(\delta R)^{\sigma-1}} \right)^{1/\sigma} + 1}.
$$

(32)

Because we have $\left( \frac{\pi}{\pi-\pi} + \frac{2(1-\pi)}{(2-\pi)(\delta R)^{\sigma-1}} \right)^{1/\sigma} > 1$ from the proof of Lemma 5, we have $c^H > y$.

The following is the proof for $c^I > y$. As in the incentive compatibility constraint in Eq. (8) where $\theta = 1$, we have

$$
\pi \left[ \frac{u(c^I)}{2} + \frac{u(2y-c^I)}{2} \right] + (1-\pi)u(c^I) = \pi u(\delta(2y-c^I)R) + (1-\pi)u(\delta yR).
$$

(33)

The left side is increasing in $c^I$ while the right side is decreasing in $c^I$ around $c^I = y$. Therefore, it is sufficient to show that by plugging $c^I$ into Eq. (33), equality in Eq. (33) is not satisfied in a way that the value of the left side is smaller than the value of the right side. Plugging in $c^I = y$, the left side becomes $u(y)$ while the right side becomes $u(y)\delta R$. Therefore, we have $c^I > y$.

**F. Proof of Proposition 2**

**Case where** $\sigma \in (1, 2)$: From Lemmas 1 and 2, we know that both $c^R$ and $c^I$ are strictly decreasing with a decreased $\theta$. As shown in Lemma 5, if $\sigma < 2$, we have $c^R < c^I$ for all $\theta \in (0, 1]$. As shown in Shell and Zhang (2018), if $c^R < c^H < c^I$, the optimal bank contract $c^*$ would be a value between $c^R$ and $c^H$, and the bank would tolerate a bank run with a small probability in equilibrium. If $c^R < c^I < c^H$, the
optimal bank contract $c^*$ would be the same as $c^I$, and the bank would tolerate a bank run in equilibrium.

As in Eq. (5), we have

$$c^R = 2y/\left(\left(\frac{2}{(\theta\delta R)^{\sigma-1}} - 1\right)^{1/(\sigma-1)} + 1\right).$$

From Eq. (34), we know that $c^R$ converges to zero as $\theta$ converges to zero. Thus, there exists $\theta^L \in (0, 1)$ such that for $\theta < \theta^L$, we have $c^R < y$, which implies that the demand deposit system cannot provide higher welfare than autarky. If $\sigma \in (1, 2)$, there exist $\theta_L, \theta_U \in (0, 1)$ and $\theta_L < \theta_U$ such that for any $\theta \in (\theta_L, \theta_U)$, the optimal contract $c^*(s)$ tolerates a run for a small value of $s$.

Case where $\sigma \in (2, \infty)$: To prove the existence of $\theta_B \in (0, 1)$, such that for any $\theta < \theta_B$, the banking contact cannot provide a higher welfare than autarky, we need to show that (a) $c^I$ is strictly increasing in $\theta$, and (b) $c^R$ can be lower than $y$ for some values of $\theta$. From the proof of Lemma 2 (in Appendix C), we have

$$\pi \left[ \frac{u(c^I)}{2} + \frac{u(2y - c^I)}{2} \right] + (1-\pi)u(c^I) = \pi u(\theta \delta(2y - c^I)R) + (1-\pi)u(\theta \delta y R).$$

and

$$\frac{dc^I}{d\theta} = \frac{\pi u'(\theta \delta(2y - c^I)R) \delta(2y - c^I)R}{\frac{1}{2}u'(c^I) - \frac{1}{2}u'(2y - c^I) + (1-\pi)u'(c^I) + \pi u'(\theta \delta(2y - c^I)R) \theta \delta R} > 0,$$

which implies that $c^I$ is strictly increasing in $\theta$. The following is the proof of (b). The left side in Eq. (35) is increasing in $c^I$ around $c^I = y$, while the right side in Eq. (35) is increasing in $\theta$. This implies that for a lower value of $\theta$, $c^I$ can be smaller than $y$. Specifically, Eq. (35) implies that if $\theta < 1/ (\delta R)$, we have $c^I < y$. Thus, if $\sigma \in (2, \infty)$, there exists $\theta_B \in (0, 1)$ such that for any $\theta < \theta_B$, the banking contact cannot provide higher welfare than autarky.

G. Proof of Proposition 3

For areas I (lower range of $\theta$) and III (higher range of $\theta$), we can prove the proposition from the fact that as $\theta$ decreases, both $c^I$ and $c^R$ decrease. We know the optimal contract $c^*$ must be smaller than or equal to $c^I$. In the proof of Proposition 2, we show that for a certain value of $\theta$, $c^I$ is the same as $y$. In this case, the ex-ante
welfare under the banking contract is smaller than the welfare under autarky, which implies that the consumer has no incentive to deposit.

The following is the proof for area II in which the optimal bank contract is affected by $\theta$. Based on Eq. (14), the normative preferences through the banking service can be written as

$$W^N(c; s) = \begin{cases} \hat{W}(c/\theta) & \text{if } c \leq c^R \\ (1 - s)\hat{W}(c/\theta) + sW^{\text{run}}(c/\theta) & \text{if } c^R < c \leq c^I \end{cases}$$

We can prove Proposition 3 in two separate cases. The first case is when there is no bank run (i.e., $\sigma > 2$ or $s > s^0$). In this case, it is trivial that as $\theta$ decreases, $c^I$ decreases, so $\hat{W}(c^I/\theta)$ decreases in the range of $y < c^I < c^H$. Because there is no bank run, $\hat{W}(c^I/\theta)$ is the welfare under banking services, i.e., $W^N(c^I; s) = \hat{W}(c^I/\theta)$. In the range of $c^I \in (y, c^H)$, $\hat{W}(c^I/\theta)$ strictly decreases.

In the case where $\sigma \in (1, 2)$, there can be a run equilibrium with a small probability of $s$. If the incentive compatibility constraint is binding, the optimal contract $c^*$ when the bank tolerates a bank run is the same as $c^I$. In this case, we have $c^* = c^I$ so the normative utility is strictly decreasing in a decreased $\theta$. If the incentive compatibility constraint is not binding, for a small positive propensity to run, the optimal overall contract balances the welfare when a run occurs and when a run does not occur (see Shell and Zhang 2018). In this case, the probability of a run affects the optimal contract. Even in this case, the welfare strictly decreases with present bias because (1) the optimal contract $c^*$ is between $(c^R, c^H)$ and (2) the welfare function is strictly increasing in $c \in (c^R, c^H)$. Therefore, $c^*$ is decreasing as $c^R$ decreases. This implies that $c^*$ is decreasing by a decreased $\theta$, which, in turn, implies that the normative welfare decreases by the lower $\theta$.

H. Proof of Lemma 4

Because the period-0 life-time utility in Eq. (17) is homothetic, with the variation of $\theta$ the utility is still affinely equivalent. Therefore, $\theta$ cannot affect the first-best allocations. From Eq. (19), we know that as $\theta \to 0$, for any value of $x_2(N - 1, d)$ (equivalently, for any value of $c_2(N - 1, d)$), the inventive compatibility constraint would be violated. Similarly, from Eq. (18), as $\theta \to 0$, for any value of $x_2(N - 1, d)$, the bank run condition would be satisfied. Therefore, there exists $\theta' \in (0, 1]$ such that for any value of $\theta < \theta'$, at the first-best allocation, $\{c^H_1(z, d), c^H_2(\alpha_1, d)\}$, the incentive compatibility in Eq. (19) is violated or the bank run condition is satisfied.
I. Proof of Proposition 4

First, as shown in Peck and Setayesh (2019), in the pre-deposit game, the bank would always choose $d = 1$ as long as the return, $R$, is constant for both the bank and asset markets. This is because the partial deposit $d < 1$ does not change the first-best allocations but can make both incentive and bank-run conditions tighter. Therefore, both the bank and the consumers have no incentive to choose partial deposits.

For a sufficiently low level of $\theta$, for the bank market to exist, $x_2(N - 1, d)$ should be very large to satisfy the incentive compatibility constraint. However, an increase in $x_2(N - 1, d)$ decreases the $\tilde{W}(c, d)$ by the concavity property of the function. Therefore, applying the same logic in the two-consumer model, with a very low level of $\theta$, it is impossible for the bank to design the optimal contract (1) to create higher welfare than that of autarky and (2) to prevent bank runs.

J. An example

We assume that the parameters are

$$\pi = 1/5; \sigma = 1.7; R = 6; y = 1, \delta = 1.$$  

The parameters will be fixed throughout the example. If $\theta > 0.217$ in the example, we have $c^H > c^R$ and there would be no run equilibrium in the optimal banking contract, as shown in Proposition 1. However, we assume that the consumer’s decision follows hyperbolic discounting. Specifically, if we assume that the present bias parameter is

$$\theta = 0.2,$$

we have

$$c^H = 1.2883; c^R = 1.1932; c^I = 1.1984$$

In this case, we have $c^H < c^R < c^I$, which implies that the bank tolerates runs in equilibrium as shown in Proposition 2. Where $\theta = 0.2$, we can calculate the following welfare values from Eqs. (3), (9) and (11):

$$W_{\theta=0.5}^{\text{Autarky}} = -2.8324; \quad \tilde{W}_{\theta=0.2}(c^I) = -1.8414$$

$$\tilde{W}_{\theta=0.2}(c^R) = -1.8420; \quad W_{\theta=0.2}^{\text{run}}(c^I) = -4.5141.$$
Figure 3: Present bias and banking contract

From Eq. (12), we have the threshold probability of a run determined by the following equation:

\[
(1 - s^0)\hat{W}_{\theta=0.2}(c^I) + s^0 W_{\theta=0.2}^{\text{run}}(c^I) = \hat{W}_{\theta=0.2}(c^R),
\]

which is equivalent to

\[
s^0 = \frac{\hat{W}_{\theta=0.2}(c^I) - \hat{W}_{\theta=0.2}(c^R)}{W_{\theta=0.2}(c^I) - W_{\theta=0.2}^{\text{run}}(c^I)} = \frac{-1.8414 - (-1.8420)}{-1.8414 - (-4.5141)} = 0.0225%.
\]

which means that if \( s < s^0 = 0.0225\% \), the optimal contract tolerates a run.

For a very low value of \( \theta = 0.1 \), we have

\[
\hat{c} = 1.2883; c^R = 0.5837; c^I = 0.5975,
\]

where \( c^I \) is lower than \( y (= 1) \), which implies that the banking service cannot result in higher welfare than autarky, even if there is no danger of a bank run. Therefore, the consumers will choose autarky.

References


