Comparative Advantage and Strategic Specialization

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Abstract
This paper shows that a strong comparative advantage is necessary for free trade and specialization in a 2×2 symmetric Ricardian model to be achieved in a Nash equilibrium. Governments strategically control labor distribution across industries, and representative agents maximize Cobb-Douglas utilities. A Nash-equilibrium with complete specialization is achieved if and only if relative productivity exceeds a key value of 3, which is considered a very large number based on previous empirical studies. This paper also introduces a two-stage game where each government chooses labor distribution first and then tariffs. In this two-stage game, complete specialization is never achieved for any relative productivity level. Finally, by generalizing the Cobb-Douglas model into CES preferences, I show that if immiserizing growth effects exist, complete specialization could not be achieved for any level of relative productivity.

Keywords: Comparative advantage; The terms of trade; Immiserizing growth; Specialization; Relative productivity; Tariff

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1 Introduction

The two most important questions in international trade are the normative question of “why should nations pursue free trade?” and the positive question of “do nations have incentives to pursue free trade?” The former question is well explained by David Ricardo, in that comparative advantage is the source of mutual benefits from world trade. This paper indicates that the answer to the second question also relies on comparative advantage; a strong comparative advantage is necessary for nations to strategically pursue free trade in a Ricardian model. Defining the degree of comparative advantage as the relative productivity between more and less competitive industries in a symmetric Ricardian model with Cobb-Douglas preferences, this paper shows that a Nash-equilibrium with complete specialization is achieved if and only if the measure exceeds a key value of 3, a very large number based on previous empirical studies. This paper also shows that complete specialization cannot be achieved if the government implements import tariffs in addition to labor distribution policies, or if there exist immiserizing growth effects.

A rational government wants to manipulate the terms of trade by protecting the less competitive sector and controlling the more competitive one. However, protectionism under strong comparative advantage results in a decrease in the domestic production size and consequently a decrease in its welfare. Therefore, a strong mutual comparative advantage should be present in order to achieve free trade. Otherwise, the government would use trade policies such as export subsidies or import tariffs. Costinot, Donaldson, Vogel and Werning (CDVW, 2015) explored the relationship between comparative advantage and optimal trade taxes. Their optimal trade taxes aim to control the production size of more competitive sectors with comparative advantage to manipulate the terms of trade.1 The difference between CDVW and this

1Brunnermeier and Sannikov (2015) also indicated that government’s capital control in undercapitalized countries can recover the optimal terms of trade and, therefore, improve welfare because firms in those countries do not internalize the negative effects of too high
paper is that the former considered import tariff and export subsidy as their strategies with a passive foreign government whereas the latter considers governments’ intervention in labor distribution in a Nash equilibrium.

Even though this paper assumes that governments directly control the labor distribution across industries rather than using indirect tools such as trade taxes, both CDVW and this paper are based on the same main logic that welfare can be improved by manipulating the terms of trade. There are many examples of policies that effectively reallocate labor towards low productivity section. Governments use various types of assistance policies such as domestic subsidies, antidumping laws, and industry bailouts to help less competitive domestic industries. These polices potentially reallocate the labor distribution from high to low productive sectors.

The main model of this paper uses a simplified framework of Ricardian models described in Dornbusch, Fischer, and Samuelson (1977) where Cobb-Douglas utility functions are used and the number of goods is two. In this paper, a representative agent in each country has a Cobb-Douglas utility function, which is considered to have a “medium” value of elasticity of substitution. The elasticity of substitution derived from the utility function is the main factor in determining the equilibrium prices, or equivalently, the terms of trade. To maximize the utility of the representative agent, each government strategically determines the quantities of labor in the two industries.

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2 Bagwell and Staiger (2011) and Broda et al. (2008) suggested empirical evidences that countries use optimal tariffs in order to manipulate the terms of trade.


4 Many research papers have shown that the existence of protectionism such as tariffs, quota, and environmental regulations is the state of Nash equilibrium because countries try to improve the terms of trade through protectionism. For Nash tariff wars, see Johnson (1951), Gorman (1958), Kemn and Riezman (1988), Gros (1987), Syropoulos (2002), Zissimos (2009) and Ogawa (2012). For Nash quota wars, see Rodriguez (1974) and Tower (1975). For strategic environmental regulations, see Barrett (1994), Kennedy (1994), Burguet and Sempere (2003) and Bhattacharya and Pal (2010).
A government’s incentive for specialization primarily depends on how much the two countries have a mutual comparative advantage. To define a measure of comparative advantage, this paper introduces a symmetric model in which one country’s labor productivity of its more (less) competitive industry is equal to the other country’s labor productivity in the more (less) competitive industry. With the help of the symmetric structure, the measure of comparative advantage is defined as the ratio of higher labor productivity to lower labor productivity, which is equal to the relative autarky price. The main results are that (1) the degree of specialization strictly increases in comparative advantage, and (2) only if comparative advantage exceeds or equals to the number 3 would a complete specialization be achieved in a Nash equilibrium.

The value of relative productivity 3 is a very large number based on previous empirical evidence. Costinot, Donaldson, and Komunjer (CDK, 2012) measured relative productivities across 21 countries and 13 industries based on the year 1997.\(^5\) Normalizing all industries to the U.S. and in all countries for the food industry, there is no single value of relative productivity among 260 values (20 countries \(\times\) 13 industries) that is greater than 3 or less than 1/3. Among the 260 values, 253 of them are in the range of [0.5,2]. The maximum is 2.74, which is the relative productivity of the Korean chemicals industry relative to that of the U.S. The minimum value is 0.49, which is the relative productivity of the Hungarian transport industry relative to that of the U.S. Considering that 3 is an unrealistically large value of relative productivity, the main result in this paper may imply that it would be hard

\(^{5}\)The list of 13 industries is Food, Textiles, Wood, Paper, Fuel, Chemicals, Plastic, Minerals, Metals, Machinery, Electrical Transport and Manufacturing not elsewhere classified. The list of 21 countries is Australia, Belgium–Luxembourg, Czech Republic, Denmark, Spain, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, the Netherlands, Poland, Portugal, Slovakia, Sweden, U.K., and U.S. The measures of relative productivity are computed using relative producer prices from the Groningen Growth and Development Centre (GGDC) Productivity Level Database. See Inklaar and Timmer (2008) for details.
for countries to agree on free trade and complete specialization without any international organizations.\textsuperscript{6}

The model in this paper is closely related to the theory of immiserizing growth developed by Bhagwati (1958), which indicates that export-biased growth might deteriorate the terms of trade sufficiently such as to decrease the welfare of the economy.\textsuperscript{7} In the context of the Ricardian model, export-biased growth can be interpreted as the expansion of the more competitive industry without contraction of the less competitive industry. However, the government’s action on labor distribution not only has the expansion effect (i.e., immiserizing growth effect) but also the contraction effect. Therefore, this paper investigates how each effect, separately, affects the government’s incentive to avoid complete specialization.

It is well-known that a lower elasticity of substitution increases the possibility of having immiserizing growth effects (see Bhagwati, Brecher and Hatta (1983, 1984)). Therefore, I extend the baseline model by including a CES utility function with a general elasticity-of-substitution parameter. With the extended model, I show that if the elasticity of substitution is lower than 0.5, which is the condition for the existence of immiserizing growth effect, complete specialization could never be achieved for any value of relative productivity. Although this paper shows that there is no immiserizing growth effect with Cobb-Douglas utility functions, the other effects (including the direct effect and terms-of-trade effects from the contraction) prevents complete specialization if the relative productivity is smaller than 3.

This paper also investigates how Nash equilibrium specialization is determined when governments are allowed to implement import tariffs. Specifically, I introduce a two-stage game where each government first strategically

\textsuperscript{6}Depending on the choice of a country for normalization rather than the U.S., some relative productivities might exceed 3. However, considering U.S. markets share the largest portion in world economy, using the U.S. as the standard country would be most appropriate.

\textsuperscript{7}Also see Bhagwati and Johnson (1960), Bhagwati (1968, 1969) and Bhagwati et al. (1984).
chooses labor distribution and then optimal import tariffs. This paper shows that complete specialization can never be achieved in such a two-stage game. This result of missing complete specialization is closely related to Kennan and Riezman (1988, 1990), where agents are endowed with goods. The two-stage game in this paper reproduces the results of Kennan and Riezman (1990), where the optimal tariffs are a function of the endowments. Kennan and Riezman (1990) show that a more equal (asymmetrical) distribution of endowments between the two sectors results in lower (higher) tariffs and thus higher (lower) utility in a Nash tariff war. In the two-stage game, “endowments” are endogenously chosen by the government’s strategic action on labor distribution. Governments know that if they choose more specialized labor distribution, which implies a more asymmetrical endowment distribution in Kennan and Riezman (1990)’s model, it will end up with the Nash equilibrium with higher tariffs and thus lower utilities. To avoid this destructive tariff war under asymmetrical endowment distribution, governments would have incentives to avoid specialization of labor distribution in the two-stage game. The main results of this paper support this logic, as it shows that the degree of specialization under the two-stage game is strictly lower than that under the one-stage game.

The rest of the paper proceeds as follows. Section 2 introduces the Ricardian model with Cobb-Douglas utility. Section 3 derives the best response functions of the labor inputs. Section 4 discusses how the degree of comparative advantage affects the Nash equilibrium trade specialization. Section 5 introduces a two-stage game where each government strategically chooses labor distribution first and then tariffs. Section 6 investigates how the theory of immiserizing growth is related to the main model of this paper. Section 7 concludes the paper.
## 2 The Model

There are two countries \( A \) and \( B \) and two goods \( x \) and \( y \). The labor productivities of good \( x \) in countries \( A \) and \( B \) are \( \sqrt{k} \) and \( 1/\sqrt{k} \), respectively. The labor productivities of good \( y \) in countries \( A \) and \( B \) are \( 1/\sqrt{k} \) and \( \sqrt{k} \), respectively. The two countries are endowed with the same amount of labor unit 1. They can invest the labor to produce either good \( x \) or good \( y \). The country A’s strategy \( a \in [0, 1] \) represents the amount of labor units invested on good \( x \) while Country B’s strategy \( b \in [0, 1] \) represents the amount of labor units invested on good \( y \). The production plan in country \( A \) and \( B \) is shown in Table 1.

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<tr>
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<th>Good ( x )</th>
<th>Good ( y )</th>
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<tbody>
<tr>
<td>Country ( A )</td>
<td>( a\sqrt{k} )</td>
<td>( (1-a)/\sqrt{k} )</td>
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<tr>
<td>Country ( B )</td>
<td>( (1-b)/\sqrt{k} )</td>
<td>( b\sqrt{k} )</td>
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Table 1. The amount of production

Without loss of generality, we assume that \( k \geq 1 \). Where \( k = 1 \), the labor productivities of two goods in both countries are identical. Where \( k \) is strictly greater than 1, the absolute and comparative advantages across the two countries arise. Country \( A \) has \( k \) times higher labor productivity in producing good \( x \) than Country \( B \) while Country \( B \) has \( k \) times higher labor productivity in producing good \( y \) than Country \( A \). Therefore, we can denote \( k \) as the Degree of Comparative Advantage (DCA).\(^8\) For \( k > 1 \), complete specialization is achieved if two countries’ strategies \( a \) and \( b \) are both equal to 1.

\(^8\)Once the representative agent’s smooth preferences are revealed, comparative advantage can be interpreted as differences in relative autarky prices between the two countries. Because of Cobb-Douglas utility functions, the measure of comparative advantage based on relative autarky prices is identical to that based on relative productivity.
The representative agent’s utility function in each country follows Cobb-Douglas preferences:

\[ U_A(x_A, y_A) = \sqrt{x_A y_A} \quad \text{and} \quad U_B(x_B, y_B) = \sqrt{x_B y_B}. \]

3 The Best Response Functions

Given the two countries’ strategies, \(a\) and \(b\), country A’s maximization problem is given by

\[
\max_{x_A, y_A} U_A(x_A, y_A),
\]

subject to \(px_A + qy_A \leq a\sqrt{k}p + \frac{1 - a}{\sqrt{k}}q, \tag{1}\]

where \(p\) and \(q\) are prices of good \(x\) and good \(y\), respectively. From Eq. (1), we can derive country A’s indirect utility function as:

\[
I_A(p, q; a) = \frac{a\sqrt{k}p + \frac{1 - a}{\sqrt{k}}q}{\sqrt{2pq}} = a\sqrt{\frac{k}{2}}\sqrt{\frac{p}{q} + \frac{1 - a}{\sqrt{2k}}\sqrt{\frac{q}{p}}}. \tag{2}\]

By the market clearing condition, we can derive the relative price as:

\[
\frac{p}{q} = \frac{\frac{1 - a}{\sqrt{k}} + b\sqrt{k}}{a\sqrt{k} + \frac{1 - b}{\sqrt{k}}}. \tag{3}\]

Plugging Eq. (3) into Eq. (2), we get the Country A’s utility as a function of \(a\) and \(b\):

\[
I_A(a|b) = a\sqrt{\frac{k}{2}}\sqrt{\frac{\frac{1 - a}{\sqrt{k}} + b\sqrt{k}}{a\sqrt{k} + \frac{1 - b}{\sqrt{k}}} + \frac{1 - a}{\sqrt{2k}}\sqrt{\frac{a\sqrt{k} + \frac{1 - b}{\sqrt{k}}}{\frac{1 - a}{\sqrt{k}} + b\sqrt{k}}}}. \tag{4}\]

The function \(I_A(a|b)\) has the following properties:

i. \(I_A''(a|b) < 0\) for all \(a, b \in [0, 1]\). (See the proof in Appendix A.)
ii. $I'_A(0|b) > 0$ for all $b \in [0,1]$. (See the proof in Appendix B.)

iii. $I'_A(1|b) < 0$ for all $b \in [0,1]$, if $k < 3$. (See the proof in Appendix C.)

iv. $I'_A(1|1) > 0$, if $k > 3$. (See the proof in Appendix D.)

v. $I'_A(1|1) = 0$, if $k = 3$. (See the proof in Appendix D.)

where $I'_A(a|b) = \frac{\partial I_A(a|b)}{\partial a}$ and $I''_A(a|b) = \frac{\partial^2 I_A(a|b)}{\partial a^2}$.

Property (i) is necessary to prove the existence of the best response function $a(b)$. Property (i) implies that for all $b \in [0,1]$, $I_A(a|b)$ is either strictly monotone in $a \in [0,1]$ or has an interior maximum value. In either case, for any $b \in [0,1]$, there exists a unique value of $a \in [0,1]$ which maximizes $I_A(a|b)$.

From properties (i), (ii) and (iii), we know that the image of the best response function excludes 1 if $k < 3$. This implies that Country $A$ ($B$) has no incentive to achieve the complete specialization no matter what Country $B$ ($A$)'s labor distribution is if the degree of comparative advantage is small enough, i.e., $k < 3$.

Properties (i) and (iv) imply that if $k \geq 3$ the best response function has a maximum value 1 if $b = 1$. This also implies that Country $A$ ($B$) has an incentive to achieve complete specialization if the degree of comparative advantage $k$ is greater or equal to 3 and Country $B$ ($A$) has achieved complete specialization.

Finally, property (v) implies that where $k = 3$, Country $A$ ($B$) has an incentive to achieve complete specialization if and only if Country $B$ ($A$) does.

4 The Nash Equilibrium

Because of the symmetric structure between the two countries, the economy has a Nash equilibrium where $a = b$. The Nash equilibrium $(a^*, b^*)$ satisfies
the following equation:

\[ I_A'(a^*|a^*) = I_A'(b^*|b^*) = I_B'(a^*|a^*) = I_B'(b^*|b^*) = 0. \]  

(5)

Solving Eq. (5), we have\(^9\)

\[ a^* = b^* = \frac{3k - 1}{6k - 1 - k^2}, \]  

(6)

where \(k < 3\).

The next step is to verify the solution (6) is the unique and stable Nash equilibrium, which is proven with the following properties (vi)-(viii).

\textbf{vi.} If \(k < 3\), there is a unique solution \(a^*\) for the equation \(I_A'(a^*|a^*) = 0\), i.e., \(a(a^*) = a^*\). The solution is that \(a^* = (3k - 1)/(6k - 1 - k^2)\). (See the proof in Appendix E.)

\textbf{vii.} If \(k < 3\), the slope of the best response function \(a(b)\) at \((a^*, b^*) = (\frac{3k - 1}{6k - 1 - k^2}, \frac{3k - 1}{6k - 1 - k^2})\) is strictly less than 1. Therefore, the Nash equilibrium is stable. (See the proof in Appendix F.)

\textbf{viii.} If \(k \geq 3\), there is no solution for \(a^* \in [0, 1]\) for the equation \(I_A'(a^*|a^*) = 0\). (See the proof in Appendix G.)

Properties (vi) and (vii) directly show that the unique stable Nash equilibrium where \(k < 3\), is

\[ (a^*, b^*) = \left( \frac{3k - 1}{6k - 1 - k^2}, \frac{3k - 1}{6k - 1 - k^2} \right) \]  

(7)

which is strictly less than 1, implying that complete specialization is not achieved if \(k < 3\).

Where \(k \geq 3\), by property (viii), we know that the best response function \(a(b)\) does not intersect the 45 degree line for any value of \(b \in [0, 1]\). The

\(^9\)See the proof in Appendix E.
The main result of this paper is summarized in the following proposition.

**Proposition 1.** Complete specialization is achieved if and only if the Degree of Comparative Advantage (DCA) is equal to or greater than 3. Where the DCA is less than 3, the degree of specialization strictly increases in the DCA.

Figure 2 shows the relationship between the degree of (Nash equilibrium) specialization and the degree of comparative advantage. The graph shows that (1) the degree of specialization increases with the degree of comparative advantage and (2) the complete specialization is achieved where the degree of comparative advantage is greater than 3. In a closed economy, the optimal value of $a$ is 1/2 regardless of the degree of comparative advantage. Therefore, for any value of $k \in (1, \infty)$, the degree of specialization, i.e., the Nash equilibrium $a^*$ ($= b^*$) is higher in the open economy than that in the closed economy.
The reason why complete specialization is not achieved in a lower level of comparative advantage is attributed to the terms-of-trade effect. If one country (country A) specializes in the production of one good (good $x$), the other country (country B) would take advantage of this by producing both goods (goods $x$ and $y$) so that the relative price of good $y$ compared to that of good $x$ will be high due to the scarcity of good $y$. In this case, the terms of trade become favors to country B. If the degree of comparative advantage is low, this terms-of-trade effect dominates welfare loss from efficient production plan and consequently countries would choose non-specialized labor distribution.

5 Two-Stage Game with Labor Distribution Policy and Tariffs

This section incorporates the conventional trade policy, import tariffs, into the main model of this paper following Kennan and Riezman (1988,1990). Specifically, I introduce a two-stage game in which each government strate-
gically determines labor distribution first, and then determine optimal tariff policy. Once the labor distribution is fixed, this model is the same as Kennan and Riezman (1988, 1990), in which agents are endowed with goods. The main result of this section is that complete specialization in this two-stage game is never achieved for any degree of comparative advantage. This result is related to Kennan and Riezman (1990)’s finding that a more asymmetrical distribution of endowments between export and import sectors results in higher tariffs and lower utilities. Because asymmetrical distribution implies high degree of specialization in the model, governments would try to deviate from complete specialization to avoid severe tariff wars.

Given \((a, b)\), we assume that countries A and B impose proportional tariffs. Then, country A’s budget constraint is

\[
px_A + q (1 + t_A) y_A \leq pa\sqrt{k} + q (1 + t_A) \frac{1-a}{\sqrt{k}} + S_A. \tag{8}
\]

where \(t_A\) represents country A’s proportional tariff and \(S_A\) is the lump-sum subsidy. The collected tariff is rebated to the representative consumer, so we have \(S_A = q^* t_A \left( y_A^* - \frac{1-a}{\sqrt{k}} \right)\), where \(q^*\) is the equilibrium price of good \(y\) and \(y_A^*\) is country A’s equilibrium consumption of good \(y\). In the same way, country B’s budget constraint is

\[
p (1 + t_B) x_B + q y_B \leq p (1 + t_B) \frac{1-b}{\sqrt{k}} + qb\sqrt{k} + S_B. \tag{9}
\]

where \(t_B\) represents country B’s proportional tariff and \(S_B \left( = q^* t_B \left( y_B^* - \frac{1-b}{\sqrt{k}} \right) \right)\) is the lump-sum subsidy.

This section proves that under the described two-stage game, complete specialization cannot be achieved in the Nash equilibrium. By contradiction, assume that the two countries are in the state of complete specialization, i.e., \((a, b) = (1, 1)\). In this case, as shown in Kennan and Riezman (1988), the Nash equilibrium tariffs \((t_A^*, t_B^*)\) diverge to infinity, i.e., \((t_A^*, t_B^*) = (\infty, \infty)\).
Then, the consumers cannot afford to buy the imported good and, consequently, utility will converge to zero (see Appendix I for the proof).\(^\text{10}\)

If country A deviates from complete specialization, i.e., \(\alpha < 1\), the utility value under autarky will be positive since \(x_A = \alpha\), \(y_B = 1 - \alpha\), and \(I_A = \sqrt{\alpha(1 - \alpha)} > 0\). After the trade, country A can increase its utility value more than that under autarky. Therefore, when country B is in the state of complete specialization, country A has an incentive to deviate from complete specialization. Under the symmetric structure of this model, both countries have mutual incentives to deviate from complete specialization. Therefore, for any degree of comparative advantage, complete specialization could not be achieved in the two-stage game.

**Proposition 2.** In a two-stage game where each government chooses the labor distribution first and then import tariffs, complete specialization cannot be achieved in the Nash equilibrium.

This paper also performs the numerical analysis for the Nash equilibrium specialization in the two-stage game.\(^\text{11}\) Figure 3 compares the degree of specialization under the one-stage with that under the two-stage game. The numerical analysis indicates that the degree of specialization under the two-stage game is strictly lower than that of the one-stage game. This is because as country A’s degree of specialization is higher, country B’s tariff policy is more effective in decreasing the world price of country A’s export good.

\(^{10}\)The Metlzer paradox can also explain why the two countries have a mutual destructive tariff war under complete specialization. The relative domestic price of foreign good under complete specialization is

\[
\frac{q(1 + t_A)}{p} = \frac{(1 + t_A)(2 + t_B)}{2 + t_A}.
\]

(\text{It can be derived from Eq. (43) in Appendix I.}) From Eq. (10), we know that an increase in \(t_A\) increases the relative domestic price of the imported good. This is the Metlzer paradox, in which the imposition of a tariff on imports decreases the relative internal price of that good. Under the paradoxical price changes, an increase in tariff will necessarily increase real purchasing power for domestic consumers.

\(^{11}\)The Matlab code is downloadable in minwook.host22.com/code/rie2016.
Figure 3: Comparative advantage and specialization in the two-stage game

To avoid country B’s tariff attacks, country A would strategically choose less specialized industry composition. In this symmetric model, the same logic can be applied to country B. Therefore, both countries have mutual incentives to pursue lower degree of specialization.

6 Immiserizing Growth and Strategic Specialization

The model in this paper is closely related to the immiserizing growth model developed by Bhagwati (1958). The immiserizing growth effect implies that an increase in the production of exporting sectors decreases the price of the exporting goods sufficiently to decrease welfare. In the Ricardian model, an expansion in the exporting sector necessarily implies a contraction in the importing sector. Therefore, this paper investigates how the expansionary and contractionary effects separately affect welfare. I modify the indirect
utility function of country A in Eq. (2) as

$$I_A(p, q; a, a') = \frac{a\sqrt{k} p + \frac{1-a'}{\sqrt{k}} q}{\sqrt{2pq}} = a\sqrt{\frac{k}{2}} \sqrt{\frac{p}{q}} + \frac{1-a'}{\sqrt{2k}} \sqrt{\frac{q}{p}},$$

(11)

where increases in $a$ and $a'$ represent the expansion and the contraction effects, respectively. If $a' = a$, the indirect utility function in Eq. (11) is the same as that in Eq. (2).

We assume that country B is in the state of complete specialization, i.e., $b = 1$. Then, the relative price in terms of $(a, a')$ is:

$$\frac{p}{q} = \frac{1-a'}{\sqrt{ak}} + \frac{k}{a\sqrt{k}} = \frac{1-a'}{ak} + \frac{1}{a}.$$

(12)

Then, from Eqs. (11) and (12), we have

$$I_A(a, a') = \sqrt{a\sqrt{1-a'+k} \sqrt{2/\sqrt{k}} + (1-a') \sqrt{a} \sqrt{2/\sqrt{1-a'+k}}},$$

(13)

There are two terms in country A’s utility function in Eq. (13). The first term $EX(a, a')$ is the utility contribution of the exporting sector, while the second term $IM(a, a')$ is that of the importing sector. The exporting-sector contribution $EX(a, a')$ is affected by both $a$ and $a'$. By the definition of the immiserizing growth model, country A experiences the immiserizing growth effect if $\partial EX(a, a')/\partial a < 0$, which means that expansion of the exporting sector decreases welfare. With Cobb-Douglas utility function, we have $\partial EX(a, a')/\partial a > 0$ (see Eq. (14)), which implies that there is no immiserizing growth effect. $EX(a, a')$ is also affected by $a'$ because the contraction of the import sector affects the relative price of the exporting good. As $a'$ increases, the relative price $p/q$ decreases and thus, $EX(a, a')$ decreases, i.e., $\partial EX(a, a')/\partial a' < 0$. $\partial IM(a, a')/\partial a$ is equal to zero, where $(a, a') = (1, 1)$, because the domestic production of imported good where $a' = 1$ is zero.
Lastly, we have \( \partial IM(a, a')/\partial a' < 0 \) because the lower production of imported goods negatively affects utility.

Where \((a, a') = (1, 1)\), we derive the following four effects from Eq. (13):

\[
\partial EX(a, a')/\partial a = \sqrt{k}/2, \tag{14}
\]

\[
\partial EX(a, a')/\partial a' = -1/ \left(2\sqrt{k}\right), \tag{15}
\]

\[
\partial IM(a, a')/\partial a = 0, \tag{16}
\]

and

\[
\partial IM(a, a')/\partial a' = -1/\sqrt{k}. \tag{17}
\]

If the sum of the four effects from Eqs. (14-17) (i.e., \(\sqrt{k}/2 - 1/ \left(2\sqrt{k}\right) - 1/\sqrt{k}\)) is negative, country A would have an incentive to deviate from complete specialization. Similar to the result in Proposition 1, if \(k\) is smaller than 3, the sum is negative; the sum is zero if \(k = 3\); and the sum is positive if \(k < 3\).

While there is no immiserizing growth effect with Cobb-Douglas utility functions, it would be interesting to investigate the relationship between the immiserizing growth effect and strategic specialization under more general utility functions. As indicated in Bhagwati, Brecher and Hatta (1983, 1984), the immiserizing growth effect can exist for low values of elasticity of substitution between export and import goods. Therefore, in this section, we generalize the utility function with CES parameters:

\[
U_A(x_A, y_A) = \left(\frac{1}{2} \frac{\varepsilon - 1}{x_A} + \frac{1}{2} \frac{\varepsilon - 1}{y_A}\right)^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{18}
\]

and

\[
U_B(x_B, y_B) = \left(\frac{1}{2} \frac{\varepsilon - 1}{x_B} + \frac{1}{2} \frac{\varepsilon - 1}{y_B}\right)^{\frac{\varepsilon}{\varepsilon - 1}}.
\]

where \(\varepsilon\) represents the elasticity of substitution between the export and import goods. As \(\varepsilon \to 1\), the utility function converges to Cobb-Douglas utility such that \(U_A(x_A, y_A) = \sqrt{x_A y_A}\) and \(U_B(x_B, y_B) = \sqrt{x_B y_B}\). With the CES
utility function, we have the following indirect utility function and the relative price if \( \beta = 1 \):

\[
U^p(\rho, \theta; \alpha, \alpha_0) = \mu \frac{\alpha^2}{\sqrt{\rho}} + 1 - \alpha^2 \frac{\sqrt{\rho}}{\theta} \quad (19)
\]

and

\[
\rho / q = \left( \frac{1 - a'}{a\sqrt{k}} + \sqrt{k} \right)^{1/\varepsilon}. \quad (20)
\]

From Eqs. (19) and (20), we can derive country A’s utility function in terms of \((a, a')\):

\[
I_A(p, q; a, a') = a \sqrt{k} \rho \left( \frac{1}{2} + \frac{1}{2} \left( \frac{a \sqrt{k}}{1 - a' + \sqrt{k}} \right)^{1/\varepsilon} \right)^{1/(1-\varepsilon)}
\]

Under the same logic as the Cobb-Douglas utility case, we can investigate country A’s incentive to deviate from complete specialization (i.e., \((a, a') = (1, 1)\)) when country B is in the state of complete specialization, i.e., \(b = 1\). The four different effects when \((a, a') = (1, 1)\) and \(b = 1\) are as follows:
\[ \partial EX(a, a')/\partial a = \frac{(-1 + 2\varepsilon)}{2\varepsilon} \sqrt{k}, \]  
\[ \partial EX(a, a')/\partial a' = -\frac{1}{2\varepsilon \sqrt{k}} < 0, \]  
\[ \partial IM(a, a')/\partial a = 0, \]  
and
\[ \partial IM(a, a')/\partial a' = -\frac{1}{\sqrt{k}} < 0. \]

Eq. (21) indicates that there exists an immiserizing growth effect if the elasticity of substitution (\(\varepsilon\)) is smaller than or equal to 0.5.\(^{12}\)

Because the other effects in Eqs. (22-24) are smaller than or equal to zero, having immiserizing growth effects is the sufficient condition for missing complete specialization in a Nash equilibrium for any level of DCA (\(k\)). This is summarized in the following proposition.

**Proposition 3.** If the immiserizing growth effect exists (i.e., the elasticity of substitution is lower than 0.5), complete specialization is not achieved for any level of comparative advantage. (See the proof in Appendix H.)

Letting the sum of the four effects from Eqs. (22-24) be zero, we have a simple relationship between \(\varepsilon\) and \(k\), \(\varepsilon = (k + 1) / (2k - 2)\), which is plotted in Figure 4. The area below the curve indicates non-specialization parameter combinations of \((\varepsilon, k)\). For \(\varepsilon < 0.5\) a country will not fully specialize for any value of \(k\). For the Cobb-Douglas case explored in this paper, where \(\varepsilon = 1,12\)

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\(^{12}\)The Ricardian model introduced in this paper has a combination of effects that include expansion and contraction, so it is not obvious whether the condition \(\partial EX(a, a')/\partial a < 0\) clearly represents the immiserizing growth effect. To clarify this issue, it is worth presenting a simple model of endowment economy. With the same CES utility functions as in Eq. (18), country A’s endowment for good \((x, y)\) is \((\sqrt{k}a, 0)\) and country B’s endowment is \((0, \sqrt{k}a')\). In this endowment economy, there exists closed form solutions for the equilibrium allocations, and the condition for country A’s welfare to decrease as \(a\) increases is that \(\varepsilon < 0.5\) for any value of \(a' > 0, a \geq a'\), and \(k > 0\), which is the same as the condition in Eq. (21).
Figure 4: The elasticity of substitution and complete specialization

$k$ needs to be more than 3 for complete specialization. For $\varepsilon$ to $\infty$ (perfect substitutes), countries fully specialize for any $k > 1$. Moreover, the graph shows that a country will not fully specialize if the immiserizing growth effect exists (i.e., the elasticity of substitution is lower than 0.5).

7 Concluding remark

The main finding of this paper is that when the government can control labor distribution, strong comparative advantage is necessary for complete specialization to be achieved in a Nash equilibrium. This paper also investigates how immiserizing growth effects are related to incomplete specialization in a CES framework. From the stylized model, this paper shows that the existence of immiserizing growth effects is the sufficient condition for missing complete specialization. Finally, this paper introduces a two-stage game where governments choose labor distribution and then optimal tariffs. In this two-stage game, complete specialization is never achieved for any degree of comparative advantage. All of these results imply that it would be difficult for countries to agree on free trade without any help from international organizations.
The alternative explanation for incomplete specialization might be a commodity-aggregation problem. While 1 or 2 digit Standard International Trade Classification (SITC) results in smaller degree of complete specialization across countries, finer digit levels (such as 3 or 4 digits) show different patterns. For example, both the U.S. and China have strong manufacturing export industries but the U.S. is more specialized in transportation equipment while China is more specialized in electronic equipment. Another explanation for the missing complete specialization could be from the Ricardo-Viner model, in which there are immobile factor endowments such as land and capital. Different from the classical Ricardian model where labor productivity is constant, the marginal return of labor decreases as the quantity of labor input increases in the Ricardo-Viner model. This diminishing-marginal-return effect prevents countries from completely giving up their less competitive industries.

Another remaining question is how the elasticity of substitution between export and import goods affects the Nash equilibrium of specialization. The main reason why countries avoid specialization is that they want to improve the terms of trade through strategic labor distribution policy. Therefore, it is expected that for lower elasticity of substitution values, complete specialization is difficult to achieve, as shown in Section 6. However, under perfect substitutes utility, i.e., \( U(x, y) = x + y \), it is apparent that the Nash equilibrium with complete specialization would be achieved for any degree of comparative advantage, because the prices are not affected by labor distribution in this case.
Appendices

A Proof of Property (i)

$I_A(a|b)$ is given by

$$I_A(a|b) = \frac{1}{\sqrt{2}} \left( a\sqrt{k}, \begin{array}{c}
\frac{1-a}{\sqrt{k}} + b\sqrt{k} \\
\frac{1-a}{\sqrt{k}} + \frac{1-b}{\sqrt{k}}
\end{array} \right) + \left( \begin{array}{c}
1-a \\
\frac{1-a}{\sqrt{k}} + b\sqrt{k}
\end{array} \right). \tag{25}$$

The function (25) is composed of two parts: $A$ and $B$. I will show that each part is strictly concave in $a$ where $0 < a < 1$. Part $A$ in (25) can be expressed as

$$A = \sqrt{\frac{a^2 (1 + bk - a)}{a + \frac{1-b}{k}}}.$$

Replacing $1 + bk$ and $\frac{1-b}{k}$ with $t_1$ (where $1 < t_1 < 1 + k$) and $t_2$ (where $0 < t_2 < 1/k$), respectively, the second derivative of (26) with respect to $a$ is

$$\frac{\partial^2}{\partial a^2} \sqrt{\frac{a^2 (t_1 - a)}{a + t_2}} = -\frac{(t_1 + t_2) a^3 (t_2 (4t_1 - 3a) + at_1)}{4 \left( \frac{(t_1 - a^2 a^2)}{t_2 + a} \right)^{3/2} (t_2 + a)^4}. \tag{27}$$

Because the ranges for $t_1$ and $t_2$ are

$$1 < t_1 < 1 + k$$

and

$$0 < t_2 < 1/k,$$

Eq. (27) is strictly negative.
Part B in (25) can be expressed as

\[ B = \sqrt{\frac{(1 - a)^2}{1 + bk - a}}. \] (28)

Replacing \(1 + bk\) and \(\frac{1-b}{k}\) with \(t_1\) and \(t_2\), respectively, the second derivative of (28) with respect to \(a\) is

\[
\frac{\partial^2 \sqrt{\frac{(1-a)^2(a+t_2)}{t_1-a}}}{\partial a^2} = (t_1 + t_2) \left\{ 3 \left( t_2 + a \right) \left( t_1 - 1 \right) + \left( t_2 + 1 \right) \left( t_1 - a \right) \right\} \quad (29)
\]

Because \(1 < t_1 < 1+k\) and \(0 < t_2 < 1/k\), (29) is strictly negative. Therefore, we have \(I_A''(a|b) < 0\).

**B Proof of Property (ii)**

From Appendix 1, we know that \(I_A(a|b)\) can be expressed in terms of \(t_1\) and \(t_2\):

\[ I_A(a|b) = \frac{1}{\sqrt{2}} \left( \sqrt{\frac{a^2(t_1-a)}{a+t_2}} + \sqrt{\frac{(1-a)^2(a+t_2)}{t_1-a}} \right), \] (30)

where \(t_1 = 1 + bk\) and \(t_2 = \frac{1-b}{k}\).

Taking derivative of (30) with respect to \(a\), we have

\[
2\sqrt{2}I'_A(a|b) = \frac{(2t_1t_2 + t_1a - 3t_2a - 2a^2)}{(t_2+a)^2} + \frac{(t_1 + t_2 - 2t_1t_2 - 3t_1a + t_2a + 2a^2)}{(t_1-a)^2} \quad (31)
\]
Where $a = 0$, the first order condition of $I_A(a|b)$ with respect to $a$ is

$$I'_A(0, b) = \frac{2t_1(1-t_2) + t_1 + t_2}{2t_1\sqrt{\frac{t_2}{n}}} > 0,$$

which is strictly positive because $t_2 < 1$.

### C Proof of Property (iii)

From Eq. (31), we can derive $I'_A(1, b)$ as

$$I'_A(1, b) = \frac{-2(1 + t_2)^2 - 2 + t_1 - 3t_2 + 2t_1t_2}{2\sqrt{\frac{n-1}{n+1}(1 + t_2)^2}}$$

$$= \frac{t_1 - 7t_2 + 2t_1t_2 - 2t_2^2 - 4}{2\sqrt{\frac{n-1}{n+1}(1 + t_2)^2}},$$

where

$$t_1 = 1 + bk \quad \text{and} \quad t_2 = \frac{1-b}{k}.$$  

The denominator in (32) is strictly positive. Therefore, we need to show the numerator in (32) is strictly negative for all $b \in [0, 1]$ and $k \in [1, 3)$. The numerator in (32) can be expressed in terms of $b$ and $k$ as

$$h(b, k) = t_1 - 7t_2 + 2t_1t_2 - 2t_2^2 - 4$$

$$= (1 + bk) - 7 \frac{1-b}{k} + 2(1 + bk) \left( 1 - \frac{b}{k} \right) - 2 \left( \frac{1-b}{k} \right)^2 - 4$$

$$= (bk - 3) + \frac{(b-1)}{k} \left( 5 + \frac{2(1-b)}{k} - 2bk \right).$$

23
$h(b, k)$ is strictly increasing in $k$ because we have
\[
\frac{\partial h(b, k)}{\partial k} = b + \frac{5}{k^2} + \frac{4}{k^3} - \frac{5b}{k^2} - \frac{8}{k^3} + \frac{4b^2}{k^3}
= b + \frac{5}{k^2}(1 - b) + \frac{4}{k^3}(1 - b)^2 > 0.
\]
Therefore, the supreme of $h(b, k)$ is achieved at $k = 3$ where $k \in [1, 3)$. Where $k = 3$, $h(b, k)$ is
\[
h(b, 3) = \frac{64}{9}b - \frac{20}{9}b^2 - \frac{44}{9} = \frac{4}{9}(16b - 5b^2 - 11).
\] (34)

From Eq. (34), we know that $h(b, 3)$ is strictly increasing in $b \in [0, 1]$. Where $b = 1$, we have $h(1, 3) = 0$. Therefore, $h(b, k)$ is strictly negative for all $b \in [0, 1]$ and $k \in [1, 3)$.

**D Proof of Properties (iv) and (v)**

Referring Eq. (32), we have
\[
I_A'(1, b) = \frac{t_1 - 7t_2 + 2t_1t_2 - 2t_2^2 - 4}{2\sqrt{\frac{b-1}{t_2+1}}(1 + t_2)^2},
\] (35)
where
\[
t_1 = 1 + bk \quad \text{and} \quad t_2 = \frac{1 - b}{k}.
\]
Where $(a, b) = (1, 1)$, we have $t_1 = 1 + k$ and $t_2 = 0$. Thus, from Eq. (35), we obtain
\[
I_A'(1, 1) = \frac{k - 3}{2\sqrt{k}},
\] (36)
which is strictly positive if $k < 3$ and strictly negative if $k > 3$. 

24
E Proof of Property (vi)

Referring Eq. (31), we have

\[ 2\sqrt{2}I_A'(a|b) = \frac{(2t_1t_2 + t_1a - 3t_2a - 2a^2)}{(t_2 + a)^2 \sqrt{\frac{t_2 + a}{t_2 + b}}} + \frac{(t_1 + t_2 - 2t_1t_2 - 3t_1a + t_2a + 2a^2)}{(t_1 - a)^2 \sqrt{\frac{t_1 + a}{t_1 - a}}} \]

where

\[ t_1 = 1 + bk \quad \text{and} \quad t_2 = \frac{1 - b}{k}. \]

Plugging \( t_1 = 1 + ak \) and \( t_2 = \frac{1-a}{k} \), we obtain

\[ 2\sqrt{2}I_A'(a|a) = \frac{a + k - 3ak - 1}{\sqrt{k(1 - a + ak)}} + \frac{2k - 3ak + ak^2}{\sqrt{k(1 - a + ak)}} \]

\[ = \frac{a + 3k - 6ak + ak^2 - 1}{\sqrt{k(ak - a + 1)}} \] (37)

Solving the equation, \( I_A'(a*|a*) = 0 \) from Eq. (37), we have

\[ a^* = \frac{3k - 1}{6k - k^2 - 1} \] (38)

which is strictly increasing in \( k \) and reaches 1 where \( k = 3 \). Where \( k > 3 \), (38) exceeds 1.

F Proof of Property (vii)

In the Nash equilibrium, i.e., \( a(b^*) = a^* \), we have the following equation:

\[ I_A'(a(b)|b) = 0 \] (39)
Implicitly differentiating (39) with respect to \( b \), we acquire

\[
a'(b) = -\frac{\partial I'_A(a|b)/\partial b}{I'_A(a|b)}. \tag{40}
\]

Plugging \((a, b) = \frac{3k - 1}{6k - k^2 - 1}\) into (40), we have

\[
a'(b) = \frac{(1 + k)^2}{14k - k^2 - 1} < 1,
\]

which is strictly less than 1 where \( k \in [1, 3] \).

**G  Proof of Property (viii)**

Referring Eq. (38), we have

\[
a^*(k) = \frac{3k - 1}{6k - k^2 - 1},
\]

which is a unique solution for \( I'_A(a^*|a^*) = 0 \). Where \( k > 3 \), \( a^*(k) \) exceeds 1. Thus, if \( k > 3 \), there is no solution for \( a^* \) in the range of \([0, 1]\) for the equation \( I'_A(a^*|a^*) = 0 \).

**H  Proof of Proposition 2**

From Eq. (21), we know that the necessary and sufficient condition for the existence of immiserizing growth effects is that \( \varepsilon > 0.5 \). From Eqs. (21-24), we know that \( I_A \) increases as \( a \) decreases if \( a = a' = 1 \). This means that if country \( B \) is in the state of complete specialization (i.e., \( b = 1 \)), country \( A \) does not have an incentive to achieve complete specialization. The same logic is applied to country \( B \). Therefore, they have mutual incentives to deviate from complete specialization.
I Analysis of the two-stage game

In the economy described in Section 6, country A’s consumption on goods $x$ and $y$ are

$$x_A = \frac{(1 - a) + ak^2_q}{\sqrt{k^2_q} (2 + t_A)}, \quad y_A = \frac{(1 - a) + ak^2_q}{\sqrt{k} (2 + t_A)}. \quad (41)$$

From (41), country A’s indirect utility is

$$I_A \left( \frac{p}{q}, t_A, t_B; a, b \right) = \frac{(1 - a) + ak^2_q}{\sqrt{k} \sqrt{q} (2 + t_A)} \sqrt{1 + t_A}$$

$$= \frac{1 - a}{\sqrt{k} \sqrt{q} + a \sqrt{k} \sqrt{p/q}} \frac{\sqrt{1 + t_A}}{2 + t_A}. \quad (42)$$

By the market clearing conditions and maximization, the relative world price of good $x$ is

$$\frac{p}{q} = \frac{bk (2 + t_A) + (1 - a) (1 + t_A) (2 + t_B)}{ak (2 + t_B) + (1 - b) (1 + t_B) (2 + t_A)}. \quad (43)$$

Under complete specialization, i.e., $(a, b) = (1, 1)$, country A’s utility is

$$I_A = \sqrt{k} \frac{1}{2 + t_B} \sqrt{\frac{1 + t_A}{2 + t_A}}, \quad (44)$$

which is strictly increasing in $t_A$ but strictly decreasing in $t_B$. Therefore, under complete specialization, the Nash equilibrium for tariffs is \((t_A^*, t_B^*) = (\infty, \infty)\).

Now, we assume that the economy is not in the state of complete specialization. Partial differentiating $p/q$ with respect to $t_A$, we have
Figure 5: The best response functions where $k = 3$ in the two-stage game

$$\frac{\partial (p/q)}{\partial t_A} = \frac{(1 + t_B) [(1 - b)(1 + t_B) + ak(1 - a)(1 + t_B) + abk^2 - (1 - b)at_B]}{[(1 - b)(2 + t_A)(1 + t_B) - ak(2 + t_B)]^2}$$

$$= \frac{(1 - b) + ak(1 - a) + \frac{abk^2 - (1 - b)a(1 + t_B)}{2 + t_B}}{ak - \frac{(1 - b)(2 + t_A)(1 + t_B)}{(2 + t_B)^2}}. \tag{45}$$

From Eqs. (42) and (45), we can derive the best response tariff-function $t_A(t_B; a, b)$, that maximizes country A’s utility conditional on $t_B$. In the same way, we can derive country B’s best response tariff-function, $t_B(t_A; a, b)$. Plugging $t_A(t_B; a, b)$ and $t_B(t_A; a, b)$ into $I_A \left( t_B, t_A, t_B; a, b \right)$, we have country A’s indirect utility function in terms of $(a, b)$, which is denoted as $I_A^T(a; b)$. From $I_A^T(a; b)$, we can derive the best response function $a(b)$ that maximizes $I_A^T(a; b)$ given $b$. In the same way, country B’s best response function $b(a)$ can be derived. For example, where $k = 3$, the two best response functions, $a(b)$ and $b(a)$, are shown in Figure 5, where the Nash equilibrium is $(a^*, b^*) = (0.62, 0.62)$. From $a(b)$ and $b(a)$, we can derive the Nash equilibrium specialization $(a^*, b^*)$ which satisfies $a(b^*) = a^*$ and $b(a^*) = b^*$. By the symmetric structure of the model, we always have $a^* = b^*$. From the deriva-
tions of $t_A(t_B; a, b)$ and $t_B(t_A; a, b)$, I apply a numerical analysis using Matlab. The Matlab code can be downloaded in minwook.host22.com/code/rie2016.

References


